1 Question (Basis algorithm for an eigenspace, lecture14a.pdf)

$$\text{Let} \quad A := \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}.$$

The number -2 is one of the eigenvalues of A. Let W be the -2-eigenspace of A.

(i) Find a basis for W. (See the end of lecture14a.pdf for an example.)

(ii) What is the dimension of W?

2 Question (rank and dimension without row reduce, lecture14b.pdf)

Let

$$B := \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 1 \\ 7 & 14 & 1 \end{bmatrix}.$$

Explain your answers without doing any row reduce computation. (Follow Exercise 5 and 6 of lecture14b.pdf.)

(a.) Find the rank of B.

(b.) Find the dimension of the image of B.

(c.) Find the dimension of the kernel of B.

3 Question (eigenbasis and matrix multiplication, lecture15a.pdf)

Let $C = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. I have verified that $C \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = -4 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $C \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, and $C \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

(a.) Is S an eigenbasis of $\mathbb{R}^3$? Explain why or why not. (See Exercise 1 of lecture15a.pdf.)

(b.) Compute

$$C^{999} \begin{bmatrix} 10 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{999} \begin{bmatrix} 10 \\ 0 \\ 1 \end{bmatrix}.$$

Don't attempt to write out all the digits. (See Exercise 2 of lecture15a.pdf.)

4 Question (Finding eigenbases, lecture15b.pdf)

$$\text{Let} \quad A := \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}.$$

a.) Without actually finding an eigenbasis, show that A has an eigenbasis. (Use one of the theorems in lecture15b.pdf.)

b.) Okay, now find an eigenbasis of A. (Please write down the corresponding eigenvalue for each vector in your eigenbasis. You'll need it to do the next problem.)

Possible sanity check: Since an eigenbasis of A is a basis for $\mathbb{R}^2$, a concatenation of your eigenbasis should have determinant non-zero.

Possible sanity check: Since each vector in an eigenbasis of A is an eigenvector of A, it must be that $Av = cv$ for each vector v where c is the corresponding eigenvalue.

5 Question (eigenbases and diagonalization, lecture15c.pdf)

Let $A := \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$, from the previous question.

a. Use the eigenbasis of A you found in the previous question to write $A = BDB^{-1}$ **where** $\mathbf{D} := \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix}$.
Then check your work by multiplying out your factorization.

b. Compute A^{100}. (See the solutions in lecture15c.pdf for guidance.)