1 Question (lecture9a.pdf)

Let $\mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}$.

(a) Compute the length of $\mathbf{u}$ and the length of $\mathbf{v}$. (Don't approximate.)

(b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

2 Question (Projection)

(Hint: A similar computation is done in Exercise 3 of lecture9b.pdf.)

Let L be the line through the points $(0,0)$ and $(1,1)$, and let $P : \mathbb{R}^2 \to \mathbb{R}^2$ be projection onto L.

a.) Find a formula for $P\left(\begin{bmatrix} a \\ b \end{bmatrix}\right)$ where $\begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2$.

b.) Find a matrix M such that $P = T_M$.

3 Question (Not T_A for any matrix A, lecture9b.pdf)

Hint: You can either write in your own words or simply copy word-for-word the solution of Exercise 4 of Lecture 9b but change the numbers appropriately. The solution to Exercise 4 is on page 18 of the PDF file lecture9b.pdf

Let g be the function which translates all 2-vectors (points) up by 2.

a.) Give a formula for g.

b.) Show that g is not a linear transformation T_A for any matrix A.

4 Question (Properties of linear transformation, lecture10a.pdf)

Use slide 6, 7, 8, 10 in lecture10a.pdf to help you fill in the blank. There should not be letters m and n. Replace them with actual numbers!

A 2×4-matrix gives a linear transformation $T_A : \mathbb{R}^4 \to \mathbb{R}^2$.

a.) The *domain* of T_A is the set of vectors of height _____ .

b.) The *target* of T_A is the set of vectors of height _____ .

c.) T_A sends $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ to _____ .

d.) T_A *preserves addition.* This means ...

e.) T_A *preserves scalar multiplication.* This means ...

f.) T_A *preserves linear combinations.* This means ...

5 Question (Not a linear transformation, lecture10a.pdf)

Define a function $G : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$G\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 \\ y \end{bmatrix}.$$

Use the properties of linear transformation (from Lecture 10) to show that G is not a linear transformation.

Write your solution following the sample answer given in lecture10a.pdf, Exercise 1.

6 Question (A linear transformation preserves linear combinations)

If $T : \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation, and we know that

$$T\left(\begin{bmatrix}1\\4\\3\\1\end{bmatrix}\right) = \begin{bmatrix}1\\4\\3\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}4\\3\\1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\5\\1\end{bmatrix}, \qquad \text{find } T\left(\begin{bmatrix}3\\-1\\-2\\0\end{bmatrix}\right).$$

Hint: See Exercise 2 of lecture10a.pdf

7 Challenge Problem (worth 0 points, but feedback will be given)

Given vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, denote their dot product by $\mathbf{u} \cdot \mathbf{v}$ as usual.

Let $\mathbf{u} := \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ in $\mathbb{R}^3$.

Define

$$T : \mathbb{R}^3 \to \mathbb{R} \text{ by}$$

$$T(\mathbf{v}) = \mathbf{u} \cdot \mathbf{v} \text{ for all } \mathbf{v} \text{ in } \mathbb{R}^3.$$

Show that T is a linear transformation.