1 Question (def of eigenvector and eigenvalue)

a.) Below, write down the definition of an **eigenvector** of a matrix and an eigen*value* of a matrix. Explain in a way that your future self (a month from now and a year from now) would understand. (See Slide 4 lecture7b.pdf)

b.) Is it possible for an **eigenvector** to be a zero vector?

◯ Yes, is it possible. There are matrices with zero vectors as eigenvectors.
◯ No, it is not possible. There are no matrices with zero vectors as eigenvectors.

c.) Is it possible for an eigen*value* to be the number 0?

◯ Yes, is it possible. There are matrices with the number 0 as an eigenvalue.
◯ No, it is not possible. There are no matrices with the number 0 as an eigenvalue.

2 Question (eigenvectors)

Reference: All examples in lecture7b.pdf are about finding λ-eigenvectors or showing they don't exist.

Let $A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find all 3-eigenvectors of A (that is, find all nonzero vectors $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ satisfying $A\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = 3\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$), if possible. Otherwise, show that A has no 3-eigenvectors.

3 Question (eigenvectors)

Let $A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find all 1-eigenvectors of A (that is, find all nonzero vectors $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ satisfying $A\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$), if possible. Otherwise, show that A has no 1-eigenvectors.

4 Question

Let $A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Find all -1-eigenvectors of A (that is, find all nonzero vectors $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ satisfying $A\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] = -\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$), if possible. Otherwise, show that A has no -1-eigenvectors.

5 Question

Suppose A is a square matrix and we know that -2 is an eigenvalue of A. Let v denote a -2-eigenvector. (That is, suppose we know that $Av = -2v$).

a.) Show that 4 is an eigenvalue of A^2 with v as an eigenvector.

b.) Show that -8 is an eigenvalue of A^3 with v as an eigenvector.

6 Question

Let $M := \begin{bmatrix} 8 & 2 \\ -6 & 0 \end{bmatrix}$.

a.) Using the definition of characteristic polynomial from lecture8a.pdf to compute the characteristic polynomial of M as a function of x.

b.) Find the roots of this characteristic polynomial.

c.) Use your answer to find all eigenvalues of M.

7 Question

Let $B := \begin{bmatrix} -2 & -2 & 4 \\ -4 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$.

a.) Find the characteristic polynomial $p_B(x)$ of B.

b.) One of the roots of this polynomial is 3. Find the other roots of $p_B(x)$. **Hint: use long division to find the other roots.**

c.) Use your previous answer to find all the eigenvalues of B.

d.) The matrix $B = \begin{bmatrix} -2 & -2 & 4 \\ -4 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ is the same as the matrix on the previous page.

Find all eigenvectors corresponding to the eigenvalue 3.

8 Question

Let

$$M := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

a.) Using the definition of characteristic polynomial given in lecture 8a notes: lecture8a.pdf, compute the characteristic polynomial of M as a function of x.

b.) The matrix M has exactly one eigenvalue λ. What is it?

c.) Solve the matrix equation

$$\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(where λ should be replaced with your answer to the previous part) to compute all λ-eigenvectors of M.

Hint: You will need multiple parameters to describe your solution set! (Maybe use the letters r, s, t).

9 Challenge Problem (worth 0 points, will not influence course grade)

Suppose A is a 4×4 matrix. Show that the eigenvalues of A and the eigenvalues of its transpose, A^T, are exactly the same.
(Hint: Try showing that their characteristic polynomials are the same.)