Week 3 Worksheet (five problems) **Math 3333**

If you have a stylus and a tablet, you may export the PDF file to your favorite note-taking app. Otherwise, write on your own notebook paper. Put each question on its own page. (Some people also like to print the worksheet into paper and write directly on it.)

References: Lecture 2a Gaussian elimination; Lecture 2b Gaussian elimination (rank, homogeneous system)

1 Question 1

Consider the system of linear equations

$$\begin{aligned} x+2y+3z &= 4 \\ 5x+6y+7z &= 8 \\ 9x+10y &= 12. \end{aligned}$$

a. Write down the augmented matrix corresponding to the system. Then use Gaussian elimination (or any sequence of elementary row operations) to find an equivalent matrix in row-echelon form (REF).

Show your work and use the examples given in the lecture notes as a template.

An equivalent matrix in row-echelon form (REF) is ____________________.

b. Use your answer to part a to quickly determine the rank of the matrices which show up in your computation of part a.

rank: __________.

c. *Without doing any more computation*, use part b to determine the rank of the augmented matrices corresponding to the following systems of linear equations (note: each of these is from Q6 of the previous worksheet — you don't need to double check).

(*i*)

$$\begin{aligned} 5x+6y+7z &= 8 \\ x+2y+3z &= 4 \\ 9x+10y &= 12 \end{aligned}$$

(*ii*)

$$\begin{aligned} 2x+4y+6z &= 8 \\ 5x+6y+7z &= 8 \\ 9x+10y &= 12 \end{aligned}$$

(*iii*)

$$\begin{aligned} x+2y+3z &= 4 \\ 7x+10y+13z &= 16 \\ 9x+10y &= 12 \end{aligned}$$

(i) rank: __________. (ii) rank: __________. (iii) rank: __________.

2 Question 2

Consider the system (from Q1 wk02 wksheet) of two linear equations in two variables

$$\begin{aligned} x - 2y &= -1 \\ x + 2y &= 3 \end{aligned}$$

Copy the graph of the system from your previous worksheet.

a. Turn this system into an augmented matrix. Then use the technique explained in Lecture 2b to compute the rank of the augmented matrix. Show your work.

The rank of the augmented matrix is ________________.

b. (Open-ended question) Discuss with your classmates some possible guesses which explain a connection between your sketch and the rank of the augmented matrix (in part a). Below, state in complete sentences your table's guess.

Your table's guess (in complete sentences):

Explain why you think your guess is reasonable.

3 Question 3

Consider the system (from Q4 of wk02 wksheet) of two linear equations in two variables

$$\begin{aligned} -6x + 2y &= -8 \\ 3x - y &= 4 \end{aligned}$$

Copy the graph of the system from your previous worksheet.

a. Turn this system into an augmented matrix. Then use the technique explained in Lecture 2b to compute the rank of the augmented matrix. Show your work.

The rank of the augmented matrix is ________________.

b. (Open-ended question) Discuss with your classmates some possible guesses which explain a connection between your sketch and the rank of the augmented matrix (in part a). Below, state in complete sentences your table's guess.

Your table's guess (in complete sentences):

Explain why you think your guess is reasonable.

4 Question 4

(See Lecture 2b) For each of the linear systems and augmented matrices, determine wether it is homogeneous or non-homogeneous, and briefly explain why.

(a.)

$$\begin{array}{ccccccc} & & 2y & + & 3z & = 4 \\ 5x & + & & & 7z & = 0 \\ 9x & + & 10y & & & = 12 \end{array}$$

☐ Yes, it is homogeneous because ________________________________

☐ No, it is not homogeneous because ________________________________

(b.)

$$\begin{array}{ccccccc} & & 2y & + & 3z & = 0 \\ 5x & + & & & 7z & = 0 \\ 9x & + & 10y & + & \pi z & = 0 \end{array}$$

☐ Yes, it is homogeneous because ________________________________

☐ No, it is not homogeneous because ________________________________

(c.)

$$\left[\begin{array}{ccc|c} 0 & 2 & 0 & 5 \\ 1 & 3 & \pi & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

☐ Yes, it is homogeneous because ________________________________

☐ No, it is not homogeneous because ________________________________

(d.)

$$\left[\begin{array}{cccc|c} 0 & 2 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \end{array}\right]$$

☐ Yes, it is homogeneous because ________________________________

☐ No, it is not homogeneous because ________________________________

5 Question 5

***Without** doing any computation*, determine whether each of the following augmented matrices corresponds to a system which has no solution, a system which has only one solution, or a system which has infinitely many solutions. Give an explanation for each.

(In your explanation, you should copy or rephrase the three cases in Lecture 2a.)

(a.)

$$\left[\begin{array}{ccc|c} \mathbf{1} & 100 & 2 & 1 \\ 0 & \mathbf{1} & \frac{1}{7} & 0 \\ 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

i. The system has ______________________ solution/s.

ii. We know this because ____

____

(b.)

$$\left[\begin{array}{cccccc|c} \mathbf{1} & 100 & 1.2 & 0 & 2 & 0 & 1 \\ 0 & \mathbf{1} & \pi & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & \frac{1}{7} & 2 & 0 \end{array}\right]$$

i. The system has ______________________ solution/s.

ii. We know this because ____

____

(c.)

$$\left[\begin{array}{cccccc|c} \mathbf{1} & 100 & 1.2 & 0 & 2 & 0 & 1 \\ 0 & \mathbf{1} & \pi & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & \frac{1}{7} & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

i. The system has ______________________ solution/s.

ii. We know this because ____

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