Week 2 Worksheet **Math 3333**

(There are ten problems total)

Problems are designed to be started during class with classmates and the instructor. References: Lec 1a, 1b

1 Question

Consider the system of two linear equations in two variables

$$x - 2y = -1$$
$$x + 2y = 3$$

a. Is the system consistent or inconsistent? If it is consistent, find solutions to the system.

b. Each equation in the system represents a line. Sketch both lines on the same graph. (First try to do these by hand, then check with a computing tool such as desmos.com/calculator).

c. The main problem: Explain in a complete sentence (or two) a connection between part (a) and (b).

2 Question

Consider the system of two linear equations in two variables

$$-2x + y = 0$$
$$-2x + y = 3$$

a. Is the system consistent or inconsistent? If it is consistent, find solutions to the system.

b. Each equation in the system represents a line. Sketch both lines on the same graph.

c. The main problem: Explain in a complete sentence (or two) a connection between part (a) and (b).

3 Question

Determine whether the following statement is true or false. If it's false, give a counterexample showing that it's false.

> If a linear system is consistent, it must have more than one solution.

4 Question

Consider the system of two linear equations in two variables

$$\begin{aligned} -6x + 2y &= -8 \\ 3x - y &= 4 \end{aligned}$$

a. Is the system consistent or inconsistent? If it is consistent, find solutions to the system.

b. Each equation in the system represents a line. Sketch both lines on the same graph. (First try to do these by hand, then check with a computing tool such as desmos.com/calculator).

c. Explain in a complete sentence (or two) a connection between part (a) and (b).

d. In what way does this system differ from the system given in Question 1? (Answer in complete sentences.)

5 Question

Determine whether the statement is true or false. If it's false given a counterexample showing that it's false.

> If a linear system is consistent, it must have exactly one solution.

6 Question

a. Describe the following operation using full sentence/s.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 5x + 6y + 7z &= 8 \\ 9x + 10y &= 12 \end{aligned} \longmapsto \begin{aligned} 5x + 6y + 7z &= 8 \\ x + 2y + 3z &= 4 \\ 9x + 10y &= 12 \end{aligned}$$

b. Describe the following operation using complete sentence/s.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 5x + 6y + 7z &= 8 \\ 9x + 10y &= 12 \end{aligned} \longmapsto \begin{aligned} 2x + 4y + 6z &= 8 \\ 5x + 6y + 7z &= 8 \\ 9x + 10y &= 12 \end{aligned}$$

c. Describe the following operation using complete sentence/s.

$$\begin{aligned} x + 2y + 3z &= 4 \\ 5x + 6y + 7z &= 8 \\ 9x + 10y &= 12 \end{aligned} \longmapsto \begin{aligned} x + 2y + 3z &= 4 \\ 7x + 10y + 13z &= 16 \\ 9x + 10y &= 12 \end{aligned}$$

7 Question

Babysitters Annie and Bob earn a total of \$24.6 when Annie works 2 hours and Bob works 3 hours. They earn a total of \$23.90 if Annie works 3 hours and Bob works 2 hours.

a. Model this situation using a system of linear equations.

b. Solve the linear system to find the hourly rate of Annie and the hourly rate of Bob. [1] Show your work.

[1] This was a long time ago, and they are still in middle school, so their hourly rates are not very high.

8 Question

(Optional: You may use tools like desmos.com/calculator to quickly visualize your system.)

For each of the following, either write down a linear system or explain why it's impossible.

1. Write down a system of three linear equations in variables x and y which has a unique solution.

2. Write down a system of three linear equations in variables x and y which has no solution.

3. Write down a system of three linear equations in variables x and y which has exactly two solution.

9 Question

Use only the techniques covered in Lec 1a and 1b to help you complete the following problem. Show your work. [2]
Consider the system of equations

$$\begin{array}{rcrcrcrcl} x & + & 2y & - & z & - & w & = & 0 \\ & & & & z & + & 2w & = & 4 \\ -x & - & 2y & + & 2z & + & 4w & = & 5 \end{array}$$

a. Solve for the value x.

i) 1 ii) 2 iii) 3 iv) x can be any number v) -1 vi) -2

b. Solve for the value y.

i) 1 ii) 2 iii) 3 iv) y can be any number v) -1 vi) -2

[2] If you use a calculator or software to verify your answer, specify the name of the calculator/software.

10 Question

Use only the techniques covered in lecture 1a and 1b to help you complete the following problem. Show your work.

Which one of the statements below is true for the system of equations

$$\begin{cases} x + y - z = 0 \\ 3x + 11y + z = 0 \\ 2x + 4y - z = 0 \end{cases} ?$$

i. The system has a unique solution $(0, 0, 0)$.

ii. The system has a unique solution $(-3, 1, -2)$.

iii. The system has solutions of the form $(1, s, s)$, where s is arbitrary.

iv. The system has solutions of the form $(3t, -t, 2t)$, where t is arbitrary.

v. The system is inconsistent.