Textbook Sec 2.1

Exercise 2.1.3 Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 1 & 4 \end{bmatrix}$, and $E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

Compute the following (where possible).

a. $3A - 2B$

b. $5C$

c. $3E^T$

d. $B + D$

e. $4A^T - 3C$

f. $(A + C)^T$

g. $2B - 3E$

h. $A - D$

i. $(B - 2E)^T$

☐ Link to problems are on Zoom chat or Canvas > Home > week 3 board work

☐ Go to jamboard link (See Zoom chat or Canvas > Home > week 3 board work

Write / type with mouse / finger on the slide labeled by your breakout room number

Solution manual:

3. b. $5C - 5\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 15 & -5 \\ 10 & 0 \end{bmatrix}$

d. $B + D$ is not defined as B is 2×3 while D is 3×2.

f. $(A + C)^T = \begin{bmatrix} 2+3 & 1-1 \\ 0+2 & -1+0 \end{bmatrix}^T = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & 2 \\ 0 & -1 \end{bmatrix}$

h. $A - D$ is not defined as A is 2×2 while D is 3×2.

Textbook Sec 2.1

Exercise 2.1.8 Simplify the following expressions where A, B, and C are matrices.

a. $\boxed{2[9(A-B)+7(2B-A)]}$
$\cancel{-2[3(2B+A)-2(A+3B)-5(A+B)]}$

$18(A-B)+\underbrace{14(2B-A)}_{28B-14A}$

$= 18A-18B+28B-14A$

$= \boxed{4A+10B}$

b. $5[3(A-B+2C)-2(3C-B)-A]$
$+2[3(3A-B+C)+2(B-2A)-2C]$

Solution manual:

8. b. $5[3(A-B+2C)-2(3C-B)-A]+2[3(3A-B+C)+2(B-2A)-2C]$
$= 5[3A-3B+6C-6C+2B-A]+2[9A-3B+3C+2B-4A-2C]$
$= 5[2A-B]+2[5A-B+C]$
$= 10A-5B+10A-2B+2C$
$= 20A-7B+2C$

Textbook Section 2.2

Exercise 2.2.2 In each case find a vector equation that is equivalent to the given system of equations. (Do not solve the equation.)

a. $$\begin{aligned} x_1 - x_2 + 3x_3 &= 5 \\ -3x_1 + x_2 + x_3 &= -6 \\ 5x_1 - 8x_2 \quad &= 9 \end{aligned}$$

A system of three equations

b. $$\begin{aligned} x_1 - 2x_2 - x_3 + x_4 &= 5 \\ -x_1 + x_3 - 2x_4 &= -3 \\ 2x_1 - 2x_2 + 7x_3 &= 8 \\ 3x_1 - 4x_2 + 9x_3 - 2x_4 &= 12 \end{aligned}$$

$$\overbrace{x_1 \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ -8 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \\ 9 \end{bmatrix}}^{\text{A vector equation}}$$

A vector equation is ...

a linear combination of some vectors = a linear combination of some vectors

Solution manual:

2. b. $$x_1 \begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 0 \\ -2 \\ -4 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 7 \\ 9 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ -2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 8 \\ 12 \end{bmatrix}$$

Exercise 2.2.10 In each case either show that the statement is true, or give an example showing that it is false.

a. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

b. If $A\mathbf{x}$ has a zero entry, then A has a row of zeros.

c. If $A\mathbf{x} = \mathbf{0}$ where $\mathbf{x} \neq \mathbf{0}$, then $A = 0$.

d. Every linear combination of vectors in $\mathbb{R}^n$ can be written in the form $A\mathbf{x}$.

e. If $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$ in terms of its columns, and if $\mathbf{b} = 3\mathbf{a}_1 - 2\mathbf{a}_2$, then the system $A\mathbf{x} = \mathbf{b}$ has a solution.

If $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$ in terms of its columns, and if the system $A\mathbf{x} = \mathbf{b}$ has a solution, then $\mathbf{b} = s\mathbf{a}_1 + t\mathbf{a}_2$ for some s, t.

g. If A is $m \times n$ and $m < n$, then $A\mathbf{x} = \mathbf{b}$ has a solution for every column $\mathbf{b}$.

h. If $A\mathbf{x} = \mathbf{b}$ has a solution for some column $\mathbf{b}$, then it has a solution for every column $\mathbf{b}$.

i. If $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions to $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution to $A\mathbf{x} = \mathbf{0}$.

j. Let $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$ in terms of its columns. If $\mathbf{a}_3 = s\mathbf{a}_1 + t\mathbf{a}_2$, then $A\mathbf{x} = \mathbf{0}$, where $\mathbf{x} = \begin{bmatrix} s \\ t \\ -1 \end{bmatrix}$.

10. b. False. $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has a zero entry, but $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ has no zero row.

d. True. The linear combination $x_1\mathbf{a}_1 + \cdots + x_n\mathbf{a}_n$ equals $A\mathbf{x}$ where, by Theorem 2.2.1, $A = \begin{bmatrix} \mathbf{a}_1 & \cdots & \mathbf{a}_n \end{bmatrix}$ is the matrix with these vectors $\mathbf{a}_i$ as its columns.

Solution manual:

False. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ then $A\mathbf{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$, and this is not a linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ because it is not a scalar multiple of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

h. False. If $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$, there is a solution $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ for $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. But there is no solution for $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Indeed, if $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ then $x - y + z = 1$ and $-x + y - z = 0$. This is impossible.