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1 Question (lecture11a.pdf)

$$\text{Let} \quad M := \begin{bmatrix} 1 & 2 & 0 & 3 & 2 \\ 1 & 0 & 2 & -1 & 6 \\ 2 & 4 & 0 & 6 & 0 \end{bmatrix}.$$

Is $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ in $\ker(M)$?

2 Question (lecture12a.pdf)

Let

$$B := \begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \end{bmatrix}.$$

Find a spanning set for the kernel of B.

3 Question (lecture14a.pdf)

Find a basis for the kernel of

$$B := \begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \end{bmatrix}$$

4 Question (lecture12a.pdf, lecture12b.pdf)

(For a similar problem, see "SAMPLE STUDENT PROOF" for Exercise 3 of lecture12a.pdf.)

Let $V = \left\{ \begin{bmatrix} a \\ b \\ a+b \end{bmatrix} \text{ where } a, b \text{ are real numbers} \right\}$, that is, the subspace of $\mathbb{R}^3$ consisting of height-3 vectors whose 3rd entry is the sum of the first two entries.

Let

$$S := \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

a.) Show that S is a spanning set for V.
b.) Determine whether S is linearly independent or linearly dependent. Explain.
c.) Is S is a basis for V? Explain.

5 Question (lecture12a.pdf, lecture12b.pdf)

Let $V = \left\{ \begin{bmatrix} a \\ b \\ a+b \end{bmatrix} \text{ where } a, b \text{ are real numbers} \right\}$ the subspace of $\mathbb{R}^3$ consisting of height-3 vectors whose 3rd entry is the sum of the first two entries.

Find a spanning set for V that is different from the spanning set given in the previous question.

6 Question (lecture12a.pdf)

$$\text{Let } \; A := \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 5 \end{bmatrix}.$$

a. Compute $\ker(A)$, i.e., find all solutions to $A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Parametrize the solution set *using parameters s and t.*

(Recommended sanity check: Pick a nonzero vector $\mathbf{v}$ in your solution set, compute $A\mathbf{v}$, and verify that it is equal to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.)

b. Find two vectors $\mathbf{v}$ and $\mathbf{w}$ where you can rewrite your parametrization of the kernel of A as $s\mathbf{v} + t\mathbf{w}$.

c. Finish each of the following sentences.

(i) The kernel of A is equal to *the set of all linear combinations* of the vectors _____

(ii) The kernel of A is equal to the *span* of the vectors _____

(iii) The kernel of A is equal to the *image* of a matrix _____

7 Question (lecture13a.pdf)

a.) Determine whether $S := \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 7 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$ is or is **not** a basis of $\mathbb{R}^3$.
b.) Use computation (show all steps of the computation) and words (write in complete sentences).

8 Question (lecture13a.pdf)

a.) Determine whether the set of vectors $S := \left\{ \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} \right\}$ is or is *not* a basis for $\mathbb{R}^3$.

b.) Use computation (show all steps of the computation) and words (write in complete sentences).

9 Question (lecture13b.pdf)

Write down a 4×5 matrix M such that the dimension of $\mathrm{im}(M)$ is 3.

10 Question (lecture13b.pdf)

$$\text{Let} \quad M := \begin{bmatrix} 1 & 2 & 0 & 3 & 2 \\ 1 & 0 & 2 & -1 & 6 \\ 2 & 4 & 0 & 6 & 0 \end{bmatrix}.$$

a.) What is the dimension of $\mathrm{im}(M)$?
b.) Now find a basis of $\mathrm{im}(M)$.

11 Question (lecture14a.pdf)

Let B be a 4×5 matrix. After row reducing to an REF matrix, we learn that this REF matrix has 3 leading 1s.

a. What is the rank of B?

b. How many parameters are required to parametrize the kernel of B?

c. How many vectors are required to span the kernel of B?

12 Question (lecture14a.pdf)

Write down a 4×5 matrix M such that the dimension of $\ker(M)$ is 2.