Week9&10 Individual Worksheet (five problems) **Math3333**

- Due Mon, Nov 2, noon at Gradescope (moved to Fri, Nov 6)
- If you have a printer, you may print the worksheet and write directly on it. If you have a stylus, you may export the PDF file to your favorite note-taking app.
- Otherwise, write on your own notebook paper. Put each question on its own page (Q1 on its own page, Q2 on its own page, and so on).
- Submit individually by uploading a scanned PDF file on Gradescope. You are encouraged to work with your classmates in breakout rooms during class meetings.
- Ask for help during class or during office hours.

Lecture references: Lecture 10a, 10b, 11a, 11b

1 Question

a. Define a function $G : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$G\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 \\ y \end{bmatrix}.$$

Use the properties of linear transformation (from Lecture 10) to show that G is not a linear transformation.
(Write your solution following the sample answer given in lecture notes 10a, Exercise 1: egunawan.github.io/la/notes/lecture10a.pdf)

b. Warm-up: If A is a 2×2 matrix, what does Lecture 10b say about the geometric meaning of $|\det(A)|$, the absolute value of the determinant of A? Lecture 10b notes: egunawan.github.io/la/notes/lecture10b.pdf

Suppose $T_A : \mathbb{R}^2 \to \mathbb{R}^2$ acts by reflection across a line through the origin. What must $|\det(A)|$ be?

(Hint: How does reflection change areas? Imagine looking in the mirror.)

2 Question

Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the function given by

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + y \\ x + 2y \end{bmatrix}$$

a. Find a matrix A such that f is equal to the linear transformation of A (denoted by T_A).

(Reference: lecture 10b, Exercise 3: egunawan.github.io/la/notes/lecture10b.pdf. Check your answer! Is $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+y \\ x+2y \end{bmatrix}$?)

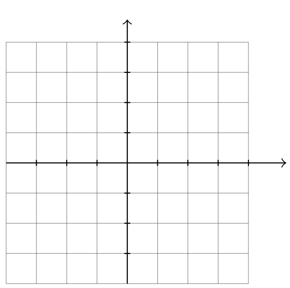
b. The matrix A has two eigenvalues, 1 and 3.

- Find all 1-eigenvectors of A, that is, find all nonzero vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ satisfying $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$.
- Find all 3-eigenvectors of A, that is, find all nonzero vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ satisfying $A\begin{bmatrix} x \\ y \end{bmatrix} = 3\begin{bmatrix} x \\ y \end{bmatrix}$.

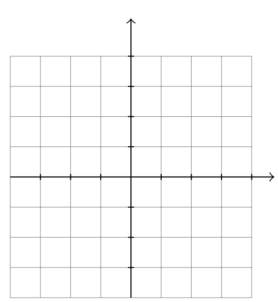
c. For each eigenvalue of A, pick one of the eigenvectors. Call your chosen 1-eigenvector v and your chosen 3-eigenvector w.

Your pick for a 1-eigenvector v: ☐. Your pick for a 3-eigenvector w: ☐

Plot v and w below



Plot $f(v)$ and $f(w)$ below



(Check that f in fact sends your v to a vector parallel to v and your w to a vector parallel to w.)

3 Question

Let

$$B := \begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \end{bmatrix}$$

Compute the kernel of B, that is, find all solutions to

$$B \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

(If you're not sure how to do this, you can follow the answer to Exercise 1 in Lecture 12a egunawan.github.io/la/notes/lecture12a.pdf)

Hint: you'll need more than one parameters. You can use a software to check your computation.

4 Question

Warm up: Review Exercise 3 of Lecture 11a notes egunawan.github.io/la/notes/lecture11a.pdf
Let

$$B := \begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \end{bmatrix} \quad \text{like in the previous problem.}$$

a. Determine whether $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ is in the image of B. Show work.

b. Determine whether $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is in the image of B. Show work.

c. Analyze the computation you did for $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then explain why every vector in the image of B must be of the form $\begin{bmatrix} t \\ -2t \end{bmatrix}$ for some number t.

(You don't need to write a full proof like in Question 5, but explain your thought process using words and math symbols.)

5 Question

To earn full credit, follow the sample answers from Exercise 6 and 7 of Lecture 11b - notes: egunawan.github.io/la/notes/lecture11b.pdf

Let V be the subset of $\mathbb{R}^3$ consisting of height-3 vectors whose 3rd entry is the sum of the first two entries. (For example, $\begin{bmatrix}1\\2\\3\end{bmatrix}$, $\begin{bmatrix}0\\2\\2\end{bmatrix}$, and $\begin{bmatrix}4\\4\\8\end{bmatrix}$ are in V). We will break down the steps of a proof that V is a subspace of $\mathbb{R}^3$.

a. Is V non-empty? Why and why not?

b. "Let v and w be in V. What does this tell you about v and w?

c. Next, give an argument that $v + w$ is in V.

d. What have you just shown about V? (Hint: use the phrase "closed under ...")

e. We need another argument to complete the proof. Fill in the blanks below to complete the proof that V is a subspace.

Next, let v be in V and let r be in $\mathbb{R}$. This means we may write v as

We then check that

$rv =$

Therefore, rv is in V, so V is ____________________________

We have shown that V is a subspace.