Week 5 Worksheet (all ten problems have been posted) **Math 3333**

- Ask for hints during class or during office hours.
- If you have a printer, you may print the worksheet. You can also export the PDF file.
- Otherwise, write on your own notebook paper.
- Put each question on its own page.
- Submit the worksheet as a group (on Gradescope as usual). If you are unable to work with your group, you can do an individual submission.

1 Question

Find all triples of numbers (a, b, c) so that $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
(Explain the steps you use to get to your answers. Use sentences and matrices.)

2 Question

Find all triples of numbers (x, y, z) so that $\begin{bmatrix} x & x \\ y & z \end{bmatrix}$ commutes with $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, that is, find all (x, y, z) so that

$$\begin{bmatrix} x & x \\ y & z \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x & x \\ y & z \end{bmatrix}.$$

3 Question

(This question is a 2D preview of a future topic)

Let $M := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

a. Compute the product $M \begin{bmatrix} x \\ y \end{bmatrix}$.

b. Plot the points $v_1 = (2, 3)$, $v_2 = (4, 6)$, $v_3 = (6, 9)$ in Cartesian coordinates:

c. Compute the following vectors:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \boxed{} \qquad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \boxed{} \qquad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 9 \end{bmatrix} = \boxed{}$$

Then, on the same graph, plot the points corresponding to the vectors computed in part (c).

d. Describe what the matrix M does to the points v_1, v_2, v_3. Use phrases like "rotation by ... degrees" or "reflection across ... line".

e. Describe what the matrix M^2 do to the points.

4 Question

(This question is a 2D preview of a future topic.)

Let $B := \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

(a) Compute the product $B \begin{bmatrix} x \\ y \end{bmatrix}$.

(b) Plot the points $v_1 = (2, 3)$, $v_2 = (4, 6)$, $v_3 = (6, 9)$ in Cartesian coordinates:

(c) Compute the following vectors:

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \boxed{} \qquad \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \boxed{} \qquad \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 9 \end{bmatrix} = \boxed{}$$

Then, on the same graph, plot the points corresponding to the vectors computed in part (c)

(d) Describe what the matrix B does to the points v_1, v_2, v_3. Use phrases like “rotation by ... degrees” or “reflection across ... line”.

(e) What does the matrix B^2 do to the points?

5 Question

Let

$$A := \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

a.) Write down the formula for the determinant $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ following Lecture 5a notes. (Use the symbols $a_{11}, a_{12}, a_{21}, a_{22}$.)

b.) Suppose that the determinant of A is non-zero. Write down the inverse of A following the formula in lecture 5b notes.

c.) Use part (b) to write down the determinant of the matrix in part (b).

d.) Compute the product of the numbers in part (a) and part (c). Is the product equal to 1?

☐ Yes, the product is equal to 1.

☐ No, the product is not equal to 1.

6 Question

(Recall two of the properties: $\det(A)\det(B) = \det(AB)$ and $\det(Id) = 1$.)

Suppose A and B are $n \times n$ matrices, and suppose A is invertible. Let $a := \det(A)$ and let $b := \det(B)$ Write the following determinants in terms of **only** a and b.

i. $\det(A^4)$

ii. $\det(ABA^{-1})$

iii. $\det(ABA^{-1}BA)$

7 Question

a. If the statement is true, explain why. If the statement is false, give a counterexample.

Statement: **If** $\det(M) \neq 0$ **and** $MA = MB$**, then** $A = B$**.**

b. If the statement is true, explain why. If the statement is false, give a counterexample.

Statement: **If** $MA = MB$**, then** $A = B$**.**

8 Question

(Scalar multiplication is defined in lecture notes 3a. Please review it.)

If the statement is true, explain why. If the statement is false, give a counterexample.

Statement: **If M is a square matrix, then $\det(-4M)$ always equals $-4\det(M)$.**

9 Question

(Matrix addition is defined in lecture notes 3a. Please review it.)

If the statement is true, explain why. If the statement is false, give a counterexample.

Statement: **If A and B are square matrices, then $\det(A+B)$ always equals $\det(A)+\det(B)$.**

10 Question

a.)
- Write down the theorem at the end of Lecture notes 5b which highlights the connection between invertibility and rank:

- Write down property i of the determinant given in Lecture notes 6a.

b.) Let M be an $n \times n$ matrix. If $\det(M) = 0$, what are the possible values for the rank of M? Use part (a) to explain your answer. (Your explanation should be in complete sentence/s.)

c.) If you know the rank of an $n \times n$ matrix M, what can you say about the determinant of M?