- If you have a printer, you may print the worksheet and write directly on it. If you have a stylus, you may export the PDF file to your favorite note-taking app.
- Otherwise, write on your own notebook paper. Put each question on its own page.
- Submit the worksheet as a group (on Gradescope as usual). If you are unable to participate during class, you may do an individual submission.
- Ask for help during class or during office hours.

Lecture references: Lecture 2a Gaussian elimination; 2b Gaussian elimination (rank, homogeneous system); 3a Matrices and vectors (addition, scalar multiplication, linear combination, transpose); 3b Matrix and vector multiplication

1 Question 1

Consider the system of linear equations from Q1 of Week1 worksheet

$$\begin{aligned} x+2y+3z &= 4 \\ 5x+6y+7z &= 8 \\ 9x+10y &= 12. \end{aligned}$$

a. Write down the augmented matrix corresponding to the system. Then use Gaussian elimination (or any sequence of elementary row operations) to find an equivalent matrix in row-echelon form (REF).

Show your work. Please completely erase any part that you don't want to be graded.

An equivalent matrix in row-echelon form (REF) is ____________________.

b. Use the answer to part a to quickly determine the rank of the matrices which show up in your computation of part a.

rank: __________.

c. *Without doing any more computation*, use part b to determine the rank of the augmented matrices corresponding to the following systems of linear equations (note: each of these is from Q1 of week1 worksheet — you don't need to double check).

(i)	(ii)	(iii)
$5x+6y+7z=8$	$2x+4y+6z=8$	$x+2y+3z=4$
$x+2y+3z=4$	$5x+6y+7z=8$	$7x+10y+13z=16$
$9x+10y=12$	$9x+10y=12$	$9x+10y=12$

(i) rank: __________. (ii) rank: __________. (iii) rank: ______________.

2 Question 2

Consider the system (from Q2 week1 worksheet) of two linear equations in two variables

$$x - 2y = -1$$
$$x + 2y = 3$$

Copy the sketch of the system from your week1 worksheet, or use desmos.com/calculator.

a. Turn this system into an augmented matrix. Then use the technique explained in Lecture 2b to compute the rank of the augmented matrix. Show your work.

The rank of the augmented matrix is ________________.

b. (Open-ended question) Discuss with your group members some possible guesses which explain a connection between your sketch and the rank of the augmented matrix (in part a). Below, state in complete sentences your group's guess.

Your group's guess (in complete sentences):

Explain why you think your guess is reasonable.

3 Question 3

Consider the system (from Q7 of week1 worksheet) of two linear equations in two variables

$$
\begin{aligned}
-6x + 2y &= -8 \\
3x - y &= 4
\end{aligned}
$$

Copy the sketch of the system from your week1 worksheet, or use desmos.com/calculator.

a. Turn this system into an augmented matrix. Then use the technique explained in Lecture 2b to compute the rank of the augmented matrix. Show your work.

The rank of the augmented matrix is ________________.

b. (Open-ended question) Discuss with your group members some possible guesses which explain a connection between your sketch and the rank of the augmented matrix (in part a). Below, state in complete sentences your group's guess.

Your group's guess (in complete sentences):

Explain why you think your guess is reasonable.

4 Question 4

(See Lecture 2b) For each of the linear systems and augmented matrices, determine wether it is homogeneous or non-homogeneous, and briefly explain why.

(a.)

$$\begin{aligned} 2y + 3z &= 4 \\ 5x + 7z &= 0 \\ 9x + 10y &= 12 \end{aligned}$$

☐ Yes, it is homogeneous because ____

☐ No, it is not homogeneous because ____

(b.)

$$\begin{aligned} 2y + 3z &= 0 \\ 5x + 7z &= 0 \\ 9x + 10y + \pi z &= 0 \end{aligned}$$

☐ Yes, it is homogeneous because ____

☐ No, it is not homogeneous because ____

(c.)

$$\left[\begin{array}{ccc|c} 0 & 2 & 0 & 5 \\ 1 & 3 & \pi & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

☐ Yes, it is homogeneous because ____

☐ No, it is not homogeneous because ____

(d.)

$$\left[\begin{array}{cccc|c} 0 & 2 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \end{array}\right]$$

☐ Yes, it is homogeneous because ____

☐ No, it is not homogeneous because ____

5 Question 5

Without doing any computation, determine (with an explanation) whether each of the following augmented matrices corresponds to a system which has no solution, a system which has only one solution, or a system which has infinitely many solutions.

(In your explanation, you should copy or rephrase the three cases in Lecture 2a)

(a.)

$$\left[\begin{array}{ccc|c} \mathbf{1} & 100 & 2 & 1 \\ 0 & \mathbf{1} & \frac{1}{7} & 0 \\ 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- The system has ______________________________ solution/s.

- We know this because ____

____

(b.)

$$\left[\begin{array}{cccccc|c} \mathbf{1} & 100 & 1.2 & 0 & 2 & 0 & 1 \\ 0 & \mathbf{1} & \pi & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & \frac{1}{7} & 2 & 0 \end{array}\right]$$

- The system has ______________________________ solution/s.

- We know this because ____

____

(c.)

$$\left[\begin{array}{cccccc|c} \mathbf{1} & 100 & 1.2 & 0 & 2 & 0 & 1 \\ 0 & \mathbf{1} & \pi & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & \frac{1}{7} & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

- The system has ______________________________ solution/s.

- We know this because ____

____

6 Question 6

Warm-up: Recall what it means for a vector to be a linear combination of other vectors (see slide no. 11 of Lecture 3a).

Write the vector $\begin{bmatrix} 24.6 \\ 23.9 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$. The scalars should be actual numbers, not just variables.

7 Question 7

Recall that a 6×4 matrix has height 6 and width 4, meaning the matrix has 6 (horizontal) rows and 4 (vertical) columns.

a. Write down a non-zero 6×4 augmented matrix which is in row-echelon form (REF) and represents a linear system which has infinitely many solutions.

b. Write down a non-zero 6×4 augmented matrix which is in row-echelon form (REF) and represents a linear system which has only one solution, or explain why it's impossible.

8 Question 8

Recall from Lecture 3a the following properties of the transpose: Let A and B denote matrices of the same size, and let r be a number.

i. If A is an $m \times n$ matrix, then A^T is an $n \times m$ matrix.

ii. $(A^T)^T = A$.

iii. $(rA)^T = rA^T$.

iv. $(A + B)^T = A^T + B^T$.

Suppose that M is a 3×3 matrix which satisfies $M = 3M^T$. Solve for M. Give your explanation following the style of Example 2.1.12 on pg 44 of the textbook.

9 Question 9

Warm-up: Review Exercise 4 in Lecture 3b: egunawan.github.io/la/notes/lecture3b.pdf.

For each of the four vectors below, determine whether it is a solution to the matrix equation

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 0 & 0 & 1 & 2 \\ -1 & -2 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}.$$

a.)

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

b.)

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

c.)

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

d.)

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

a.)	b.)	c.)	d.)
a solution/not a solution	a solution/not a solution	a solution/not a solution	a solution/not a solution

e.) Neatly write out your computation for at least one of (a)-(d) (the rest you can do on scratch paper).

You can verify your answers with a computing tool, for example, desmos.com/matrix.

10 Question 10

Compute the following product or state that it is undefined.

a)
$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 11 & 1 \\ 2 & 4 & -1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 11 & 1 \\ 2 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

a)

b)

(c) Neatly write out your computation for at least one of (a) or (b) (the other you can do on scratch paper).