Week 11 Individual Worksheet (four pages, Lecture 12a and 12b) **Math 3333**

- Due Monday, Nov 9, 2020 on Gradescope
- If you have a printer or a tablet/stylus, you may write directly on the worksheet.
- Otherwise, write on your own notebook paper. Put each question on its own page.
- Submit individually (on Gradescope). You are encouraged to work with your classmates in breakout rooms during class meetings and ask for help during class/office hours.

1 Question

Let V be the subspace of $\mathbb{R}^3$ consisting of height-3 vectors whose 3rd entry is the sum of the first two entries. For example, $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\end{bmatrix}$ are in V. (In the week9/10 worksheet, you showed that this subset is a subpsace — you don't need to show it again.) Let

$$S := \left\{ \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix} \right\}$$

Show that S is a spanning set for V. (For full credit, follow "SAMPLE STUDENT PROOF" in Exercise 3 of Lecture 12a.)

2 Question

Let

$$S := \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(from the previous question).

a. Use the computation you did earlier to determine whether S is linearly independent or linearly dependent. Explain. (You don't need to repeat the row reduce computation from the previous question.)

(For determining whether a set is linearly dependent or independent, see Lecture 12b Exercise 5 and 6, 7 in Lecture 12b.)

b. Let V be the subspace of $\mathbb{R}^3$ consisting of height-3 vectors whose 3rd entry is the sum of the first two entries (from the previous question). Use both part (a) of this page and the previous Question 1 to determine whether S is a basis for V. Give a brief explanation (write in complete sentences).

3 Question

$$\text{Let} \quad A := \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 5 \end{bmatrix}.$$

a. Compute $\ker(A)$, i.e., find all solutions to $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Then parametrize the solution set *using the parameters s and t.*

(You may use a software to row reduce only. Show your input and output.)

(Recommended: Perform a check, i.e. compute Av where v is a vector in your solution set and verify that it is equal to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.)

b. Find two vectors $\mathbf{v}$ and $\mathbf{w}$ where you can rewrite your parametrization of the kernel of A as $s\mathbf{v} + t\mathbf{w}$.

c. Finish each of the following sentences.

(i) The kernel of A is equal to *the set of all linear combinations* of the vectors ______________________

(ii) The kernel of A is equal to the *span* of the vectors ______________________

(iii) The kernel of A is equal to the *image* of the matrix ____________________ (Hint: See Exercise 2 & Fact 1 in Lecture 12a.)

4 Question

a.

$$\text{Let } A := \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 5 \end{bmatrix} \text{ (the same as in the previous question).}$$

(You have computed an REF matrix equivalent to A in the previous question. You don't need to do this again.)

How many leading 1s does your REF matrix have?

What is the rank of A?

How many parameters were required to parametrize the solution to $Ax = 0$?

(Hint/recall: when a column has **no** leading 1, you set the variable corresponding to that column to a paramater. Also, I gave you the number of parameters in the previous question.)

b. Let B be a 4×5 matrix (4 rows & 5 columns). After row reducing to an REF matrix, you count that it has 3 leading 1s.

What is the rank of B?

How many parameters are required to parametrize the kernel of B?

c. Let C be an $r \times c$ matrix and suppose C has rank k. How many vectors are required to span the kernel of C?