(There are ten problems total)

The problems are designed to be worked on in groups during class with the help of the instructor and TA, and submitted as a group. But you may do an individual submission if you are unable to participate during class.

1   Problem

a. Describe the following operation using full sentence/s.

$$
\begin{aligned}
x + 2y + 3z &= 4 \\
5x + 6y + 7z &= 8 \\
9x + 10y &= 12
\end{aligned}
\quad\longmapsto\quad
\begin{aligned}
5x + 6y + 7z &= 8 \\
x + 2y + 3z &= 4 \\
9x + 10y &= 12
\end{aligned}
$$

b. Describe the following operation using complete sentence/s.

$$
\begin{aligned}
x + 2y + 3z &= 4 \\
5x + 6y + 7z &= 8 \\
9x + 10y &= 12
\end{aligned}
\quad\longmapsto\quad
\begin{aligned}
2x + 4y + 6z &= 8 \\
5x + 6y + 7z &= 8 \\
9x + 10y &= 12
\end{aligned}
$$

c. Describe the following operation using complete sentence/s.

$$
\begin{aligned}
x + 2y + 3z &= 4 \\
5x + 6y + 7z &= 8 \\
9x + 10y &= 12
\end{aligned}
\quad\longmapsto\quad
\begin{aligned}
x + 2y + 3z &= 4 \\
7x + 10y + 13z &= 16 \\
9x + 10y &= 12
\end{aligned}
$$

2   Question 2

Consider the system of two linear equations in two variables

$$
\begin{aligned}
x - 2y &= -1 \\
x + 2y &= 3
\end{aligned}
$$

a. Is the system consistent or inconsistent? If it is consistent, find solutions to the system.

b. Each equation in the system represents a line. Sketch both lines on the same graph. (First try to do these by hand, then check with a computing tool such as desmos.com/calculator).

c. The main problem: Explain in a complete sentence (or two) a connection between part (a) and (b).

3   Question

Consider the system of two linear equations in two variables

$$
\begin{aligned}
-2x + y &= 0 \\
-2x + y &= 3
\end{aligned}
$$

a. Is the system consistent or inconsistent? If it is consistent, find solutions to the system.

b. Each equation in the system represents a line. Sketch both lines on the same graph. (First try to do these by hand, then check with a computing tool such as desmos.com/calculator).

c. The main problem: Explain in a complete sentence (or two) a connection between part (a) and (b).

4   Question

Determine whether the following statement is true or false. If it's false given an example showing that it's false.

> If a linear system is consistent, it must have more than one solution.

5   Question

Determine whether the statement is true or false. If it's false given an example showing that it's false.

> If a linear system is consistent, it must have exactly one solution.

6   Question

Babysitters Annie and Bob earn a total of \$24.6 when Annie works 2 hours and Bob works 3 hours. They earn a total of \$23.90 if Annie works 3 hours and Bob works 2 hours.

a. Model this situation using a system of linear equations.

b. Solve the linear system to find the hourly rate of Annie and the hourly rate of Bob. [1]

Show your work. You may use a calculator to do the arithmetic.

7   Question

Consider the system of two linear equations in two variables

$$-6x + 2y = -8$$
$$3x - y = 4$$

a. Is the system consistent or inconsistent? If it is consistent, find solutions to the system.

b. Each equation in the system represents a line. Sketch both lines on the same graph. (First try to do these by hand, then check with a computing tool such as desmos.com/calculator).

c. Explain in a complete sentence (or two) a connection between part (a) and (b).

d. In what way does this system differ from the system given in Question 2? (Answer in complete sentences.)

8   Question

(Optional: You may use tools like desmos.com/calculator to quickly visualize your system.)

For each of the following, either write down a linear system or explain why it's impossible.

1. Write down a system of three linear equations in variables x and y which has a unique solution.

2. Write down a system of three linear equations in variables x and y which has no solution.

3. Write down a system of three linear equations in variables x and y which has exactly two solution.

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[1]This was a long time ago, and they are still in middle school, so their hourly rates are not very high.

9   Question

Use only the techniques covered in lecture 1a and 1b to help you complete the following problem. Show your work.
[2]

Consider the system of equations

$$
\begin{array}{rrrrrrrrr}
x & + & 2y & - & z & - & w & = & 0 \\
 & & & & z & + & 2w & = & 4 \\
-x & - & 2y & + & 2z & + & 4w & = & 5 \\
\end{array}
$$

a. Solve for the value x.

   i) 1   ii) 2   iii) 3   iv) x can be any number   v) -1 vi) -2

b. Solve for the value y.

   i) 1   ii) 2   iii) 3   iv) y can be any number   v) -1 vi) -2

10   Question

Use only the techniques covered in lecture 1a and 1b to help you complete the following problem. Show your work.

Which one of the statements below is true for the system of equations

$$
\left\{
\begin{array}{rrrrrrr}
x & + & y & - & z & = & 0 \\
3x & + & 11y & + & z & = & 0 \\
2x & + & 4y & - & z & = & 0 \\
\end{array}
\right. \ ?
$$

   i. The system has a unique solution $(0, 0, 0)$.

   ii. The system has a unique solution $(-3, 1, -2)$.

   iii. The system has solutions of the form $(1, s, s)$, where s is arbitrary.

   iv. The system has solutions of the form $(3t, -t, 2t)$, where t is arbitrary.

   v. The system is inconsistent.

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[2]If you use a calculator or software to verify your answer, specify the name of the calculator/software.