9b

Let $A := \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$

Domain of T_A ?

Target of T_A ?

Find a vector v so that $T_A(v) = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

Use the fact that

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right] \xrightarrow[\text{reduce}]{\text{Row}} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

10 b

Def Saying that a function $f: \underset{(\text{domain})}{\mathbb{R}^n} \to \underset{(\text{target})}{\mathbb{R}^m}$ preserves addition means ...

Which transformations are linear transformations ?

□ $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 + y \\ 2y \end{bmatrix}$

□ $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5x + y \\ 2y \\ 4x \end{bmatrix}$

□ $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 2 \\ y \end{bmatrix}$

□ A rotation by $50°$ around the origin

□ A reflection with respect to the line $y = 4x$

□ A projection onto the line $y = 4x$

(See Exercise 2 Lec 10 a)

Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $f(v_1) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $f(v_2) = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

and $v_3 = 3v_1 - 2v_2$

Compute $f(v_3)$

10 b

What are the standard basis vectors in $\mathbb{R}^3$?

$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Compute the matrix of the linear transformation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5x+y \\ 2y \\ 4x \end{bmatrix}$$

11a (Write these definitions on a cheat sheet)

Def We say a subset S of $\mathbb{R}^n$ is closed under scalar multiplication if ...

Def
In this class, we defined a subspace V of $\mathbb{R}^n$ to be ...

If S is a subspace of $\mathbb{R}^4$, does S contain $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$?

Def The image of an $\overbrace{m}^{\#\text{ rows}} \times \overbrace{n}^{\#\text{cols}}$ matrix A is ...

Let $A := \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$

- Is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in $\operatorname{im}(A)$?

- Is $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ in $\operatorname{im}(A)$? Use the fact that $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right] \xrightarrow[\text{reduce}]{\text{Row}} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

- Is $\begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$ in $\operatorname{im}(A)$? Use the fact that $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right] \xrightarrow[\text{reduce}]{\text{Row}} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$

- Is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ in $\operatorname{im}(A)$?