Chemical Reactions

When a chemical reaction takes place a number of molecules combine to produce new molecules. Hence, when hydrogen H_2 and oxygen O_2 molecules combine, the result is water H_2O. We express this as

$$H_2 + O_2 \rightarrow H_2O$$

Individual atoms are neither created nor destroyed, so the number of hydrogen and oxygen atoms going into the reaction must equal the number coming out (in the form of water). In this case the reaction is said to be *balanced*. Note that each hydrogen molecule H_2 consists of two atoms as does each oxygen molecule O_2, while a water molecule H_2O consists of two hydrogen atoms and one oxygen atom. In the above reaction, this requires that twice as many hydrogen molecules enter the reaction; we express this as follows:

$$2H_2 + O_2 \rightarrow 2H_2O$$

This is now balanced because there are 4 hydrogen atoms and 2 oxygen atoms on each side of the reaction.

Example

Balance the following reaction for burning octane C_8H_{18} in oxygen O_2:

$$C_8H_{18} + O_2 \to CO_2 + H_2O$$

where CO_2 represents carbon dioxide.

Solution. We must find positive integers x, y, z, and w such that

$$xC_8H_{18} + yO_2 \to zCO_2 + wH_2O$$

Equating the number of carbon, hydrogen, and oxygen atoms on each side gives $8x = z$, $18x = 2w$ and $2y = 2z + w$, respectively. These can be written as a homogeneous linear system

$$\begin{aligned} 8x \quad - \quad z \quad &= 0 \\ 18x \quad &- 2w = 0 \\ 2y - 2z - \quad w \quad &= 0 \end{aligned}$$

The subspace of the solutions to this system has infinitely many solutions, requiring one parameter. For example, if we set $w = t$, we get $x = \frac{1}{9}t$, $z = \frac{8}{9}t$, $2y = \frac{16}{9}t + t = \frac{25}{9}t$. *But x, y, z, and w must be positive integers*, so the smallest value of t that eliminates fractions is 18. Hence, $x = 2$, $y = 25$, $z = 16$, and $w = 18$, and the balanced reaction is

$$2C_8H_{18} + 25O_2 \to 16CO_2 + 18H_2O$$

Sanity check: verify that this is indeed balanced.

Example

Balance the chemical reaction given below involving tin (Sn), hydrogen (H), and oxygen (0).

$$xSnO_2 + yH_2 \rightarrow zSn + wH_2O$$

Solution

Setting up a system of equations in x, y, z, w gives

$$\begin{aligned} Sn &: \quad x = z \text{ or } x - z = 0 \\ O &: \quad 2x = w \text{ or } 2x - w = 0 \\ H &: \quad 2y = 2w \text{ or } 2y - 2w = 0 \end{aligned}$$

The augmented matrix is $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right]$

Since the 4th column has no leading one, we let $w = t$. The general solution is $x = \frac{1}{2}t,\ y = t,\ z = \frac{1}{2}t,\ w = t$.

We can choose any values for $w = t$ as long as x, y, z, w are positive integers. Suppose we choose $w = 4$, then $x = 2, y = 4, z = 2$ and the balanced reaction is

$$\boxed{2Sn0_2 + 4H_2 \rightarrow 2Sn + 4H_2O}$$

Student PROBLEM 2

a. Using the methods shown above, balance the following chemical reaction. Show all work. No credit is given for a balanced reaction with no explanation.

$$CH_4 + O_2 \rightarrow CO_2 + H_2O.$$

Hint: this is the combustion of methane, CH_4. After you show work and get an answer, you can look this up.

b. Using the methods shown above, balance the following chemical reaction. Show all work. No credit is given for a balanced reaction with no explanation.

$$CO_2 + H_2O \rightarrow C_6H_{12}O_6 + O_2.$$

Hint: This reaction is called photosynthesis, a process which produces glucose ($C_6H_{12}O_6$) and oxygen. After you show work and get an answer, you can look this up.