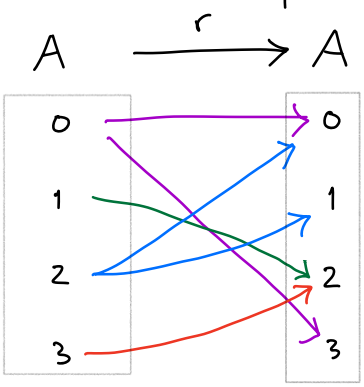
Sec 6.2 Graphs of relations on a set

Ex Let $A = \{0, 1, 2, 3\}$. Consider a relation

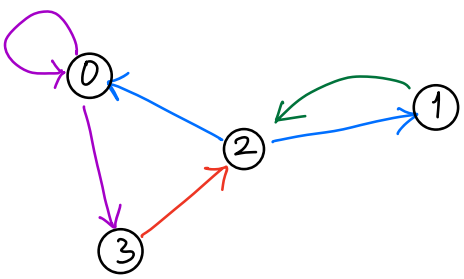
$$r = \{(0,0), (0,3), (1,2), (2,1), (3,2), (2,0)\}.$$

In Sec 6.1, we visualize r as follows:



(Two vertices per element of A)

Instead of drawing two copies of the set A, we can just draw one:



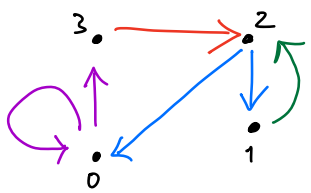
(One vertex per element of A)

- This type of graph of a relation r is called a <u>directed graph</u> or <u>digraph</u>.
- Elements of A are called the <u>vertices</u> (or <u>nodes</u> or <u>points</u>)
- Connect vertex x to vertex y with an <u>arrow</u> (or <u>edge</u>) going from x to y iff $x \; r \; y$.
- Note: we draw a "self-loop" at 0 since $0 \; r \; 0$.

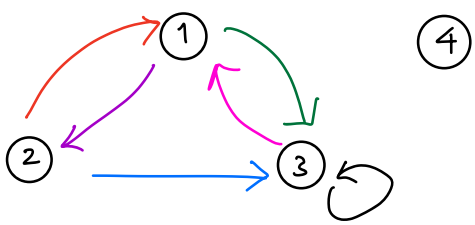
- The actual location of the vertices in a digraph doesn't matter. The location of vertices that we choose is called an <u>embedding</u> of the graph.

  Usually we try to place the vertices in such a way that the graph is easy to read.

Different embedding of the same digraph:



<u>Ex</u> Let $A = \{1, 2, 3, 4\}$. Consider the relation s on A whose digraph is below:



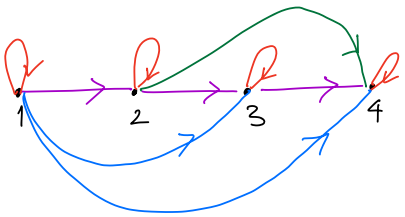
What is the relation s, as a subset of $A \times A$?

Ans: $s = \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,3)\}$

Book Exercise 1:

Let $A = \{1,2,3,4\}$, and let r be the relation $\leq$ on A.

Draw a digraph for r.



(We'll learn soon that r is an example of a partial ordering and we can draw a simpler graph)

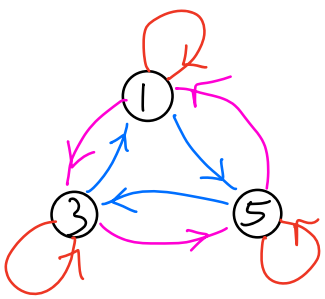
"poset"

Book Exercise 3:

Let $A = \{1,2,3,4,5\}$. Define t on A by $a\,t\,b$ if and only if $b-a$ is even.

Draw a digraph for t.



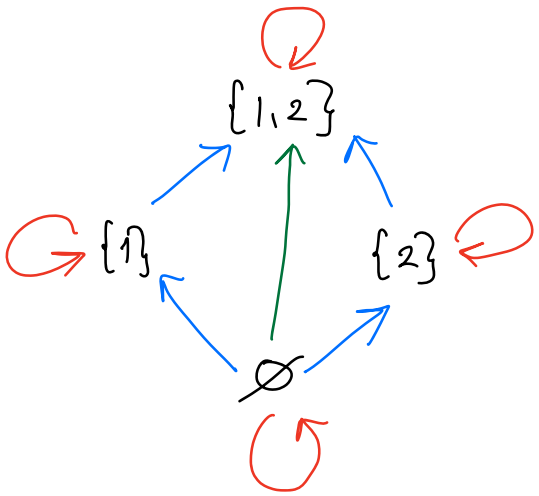


Note: $t = \{(a,b) \in A \times A \mid a,b \text{ are both even OR } a,b \text{ are both odd}\}$

$|t| = \underbrace{3 \times 3}_{\text{every tuple in } \{1,3,5\}\times\{1,3,5\}} + \underbrace{2 \times 2}_{\text{every } (a,b) \text{ such that } a,b \in \{2,4\}} = 13$

We'll learn that the way t partitions $\{1,2,3,4,5\}$ into the sets $\{1,3,5\}$ and $\{2,4\}$ is an example of the way an equivalence relation on a set splits that set into equivalence classes.

Ex (Ordering subsets of a two-element universe).
Let $A = \mathcal{P}(\{1,2\}) = \{\varnothing, \{1\}, \{2\}, \{1,2\}\}$.
Then $\subseteq$ is a relation on A. Its digraph is below:



Note
Later we'll see $\subseteq$ is a "partial ordering"
and we can visualize it by a "Hasse diagram":

