

Sec 3.4 The laws of logic

You don't need to memorize Table 3.4.3, 3.4.4
(or even write them on your cheat sheet)

but you need to be able to justify these
equivalences ($\Leftrightarrow$) and implications ($\Rightarrow$)

Ex $\neg(P \vee Q)$ is equivalent to $(\neg P) \wedge (\neg Q)$

Symbol: $\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$

Justification:

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$(\neg P) \wedge (\neg Q)$	$\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$
0	0	0	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	1	0	0	1	0	1
1	1	1	0	0	0	0	1

This shows that the proposition $\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$
is a tautology. So $\neg(P \vee Q)$ is equivalent to $(\neg P) \wedge (\neg Q)$

Note: $\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$ is called a DeMorgan's law

The other DeMorgan's law is its "dual":

$$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$$

Ex Recall 0 is a symbol for a contradiction,

1 is a symbol for tautology

Null laws: $P \wedge 0 \Leftrightarrow 0$

Justification:

P	0	$P \wedge 0$	$P \wedge 0 \Leftrightarrow 0$
0	0	0	1
1	0	0	1

$P \vee 1 \Leftrightarrow 1$

Justification:

P	1	$P \vee 1$	$P \vee 1 \Leftrightarrow 1$
0	1	1	1
1	1	1	1

Other common logic laws that we'll use when constructing an argument:

1. (Conditional Equivalence) $P \rightarrow Q$ is equivalent to $\neg P \vee Q$
 Symbol: $(P \rightarrow Q) \Leftrightarrow \neg P \vee Q$

Justification:

P	Q	$P \rightarrow Q$	$\neg P$	$\neg P \vee Q$	$(P \rightarrow Q) \Leftrightarrow \neg P \vee Q$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

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2. (Detachment, aka "modus ponens")

$P \wedge (P \rightarrow Q)$ implies Q Symbol: $P \wedge (P \rightarrow Q) \Rightarrow Q$

Justification:

P	Q	$P \rightarrow Q$	$P \wedge (P \rightarrow Q)$	$[P \wedge (P \rightarrow Q)] \rightarrow Q$
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

Longer chains of detachment are possible, e.g.

$(P \wedge (P \rightarrow Q)) \wedge (Q \rightarrow R)$ implies R

$(P \wedge (P \rightarrow Q)) \wedge ((P \wedge Q) \rightarrow R)$ implies R

3. (Chain rule) $(A \rightarrow B) \wedge (B \rightarrow C)$ implies $(A \rightarrow C)$

A	B	C	$A \rightarrow B$	$B \rightarrow C$	$(A \rightarrow B) \wedge (B \rightarrow C)$	$A \rightarrow C$	$[(A \rightarrow B) \wedge (B \rightarrow C)] \rightarrow (A \rightarrow C)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	1	1	1	1	1
1	1	0	0	0	0	0	1
1	1	1	1	1	1	1	1

Longer chains:

$$[(A \rightarrow B) \text{ and } (B \rightarrow C) \text{ and } (C \rightarrow D) \text{ and } (D \rightarrow E)] \Rightarrow (A \rightarrow E)$$

4. (Contrapositive) $P \rightarrow Q$ is equivalent to $\neg Q \rightarrow \neg P$

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \rightarrow \neg P$	$(P \rightarrow Q) \rightarrow \neg Q \rightarrow \neg P$
0	0					1
0	1					1
1	0					1
1	1					1

↑ ↑ ↑ ↑
fill in!

Exercise #2:

Show that $(P \rightarrow Q) \wedge \neg P \Rightarrow \neg Q$ is not a law of logic.

i.e. show $[(P \rightarrow Q) \wedge \neg P] \rightarrow \neg Q$ is not a tautology.

Ex (A logic puzzle by Lewis Carroll)

What conclusion can you draw from the following assumptions?

(i) All babies are illogical

(ii) Nobody is despised who can manage a crocodile

(iii) Illogical persons are despised.

Let B = "x is a baby", D = "x is despised",

C = "x can manage a crocodile",

I = "x is illogical"

Exercise Convert the three assumptions into symbolic form.

(i) $B \rightarrow I$

(ii) $C \rightarrow \neg D$

(iii) $I \rightarrow D$

Next exercise Chain $B \rightarrow I$, $C \rightarrow \neg D$, and $I \rightarrow D$ together to draw a conclusion that uses all of them.

$B \rightarrow I$ and $I \rightarrow D$ and $\underbrace{D \rightarrow \neg C}_{\text{the contrapositive of (ii)}}$

By the Chain Rule, $B \rightarrow \neg C$

i.e. If x is a baby then x cannot manage a crocodile,

i.e. No baby can manage a crocodile.