

Sec 3.2 Truth tables

Recall: Negation $\neg$
 (from Sec 3.1) Conjunction $\wedge$ "and"
 Disjunction $\vee$ "or"
 Conditional statement $P \rightarrow Q$
 Biconditional proposition $P \leftrightarrow Q$

- Suppose P, Q , and R are propositions.
- Let x be the compound proposition $(P \wedge Q) \vee (\neg Q \wedge R)$.
- We say x is a "proposition generated by p, q , and r "
 (In general, a proposition generated by a set S of propositions is any valid combination of propositions in S with conjunction, disjunction, and negation)

The truth table for x has $2^3 = 8$ rows, one row for each possible values of p, q, r :

Follow
lex
order
↓

p	q	r	$p \wedge q$	$\neg q$	$\neg q \wedge r$	$(p \wedge q) \vee (\neg q \wedge r)$
0	0	0	0	1	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	0	0	1

Hierarchy of symbols (left to right)

1. Negation $\neg$
2. Conjunction $\wedge$
3. Disjunction $\vee$
4. Conditional statement $P \rightarrow Q$
5. Biconditional proposition $P \leftrightarrow Q$

Ex $P \wedge Q \vee R$ is $(P \wedge Q) \vee R$

$\neg P \vee \neg R$ is $(\neg P) \vee (\neg R)$

$P \leftrightarrow Q \wedge R \rightarrow S$ is $P \leftrightarrow ((Q \wedge R) \rightarrow S)$

Group exercise:

Construct the truth tables of :

(a) $\neg(P \wedge Q)$

(b) $P \wedge (\neg Q)$

(c) $(P \wedge Q) \wedge R$

(d) Extra: $(P \wedge Q) \vee R$

(e) Extra: $P \wedge (Q \vee R)$

(a)

p	q	$p \wedge q$	$\neg(p \wedge q)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

(b)

p	q	$\neg q$	$p \wedge (\neg q)$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

(c)

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

Note: Putting the columns for $\neg(P \wedge Q)$ and $P \wedge (\neg Q)$ side by side ...

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg q$	$p \wedge (\neg q)$
0	0	0	1	1	0
0	1	0	1	0	0
1	0	0	1	1	1
1	1	1	0	0	0

we see that, for all propositions P and Q ,
 if $P \wedge (\neg Q)$ holds then $\neg(P \wedge Q)$ holds,
 i.e. $P \wedge (\neg Q)$ implies $\neg(P \wedge Q)$.

We write " $P \wedge (\neg Q) \Rightarrow \neg(P \wedge Q)$ "

This is called an implication & we'll discuss this in Sec 3.3.