

Sec 3.1 Propositions & Logical Operators

Propositions & Logical Operators

Def A proposition is a sentence to which one and only one of the terms true or false can be meaningfully applied.

Examples of propositions

"Four is even"

" $4 \in \{1, 3, 5\}$ "

" $43 > 21$ "

" $x^2 = y^2$ is a necessary condition for $x = y$ "

"I like discrete structures"

"The world is flat"

The most common logical operators used to combine simple statements into compound statements are:

1. and
2. or
3. not
4. if ... then
5. ... if and only if

Def Let P and Q be two propositions.

- (1) The conjunction of P and Q , denoted $P \wedge Q$, is defined by the truth table

P	Q	$P \wedge Q$	every day speech
0	0	0	P is false, Q is false, " P and Q " is false
0	1	0	P is false, Q is true, " P and Q " is false
1	0	0	P is true, Q is false, " P and Q " is false
1	1	1	P is true, Q is true, " P and Q " is true

The numbers 0 and 1 denote false and true, respectively.

How to remember the symbol $\wedge$? Like the "A" in "and"

- (2) The disjunction of P and Q , denoted $P \vee Q$, is defined by the truth table

P	Q	$P \vee Q$	every day speech
0	0	0	P is false, Q is false, " P or Q " is false
0	1	1	P is false, Q is true, " P or Q " is true
1	0	1	P is true, Q is false, " P or Q " is true
1	1	1	P is true, Q is true, " P or Q " is true

How to remember the symbol $\vee$? "vel" is the Latin word for "or"

Ex Consider the following compound proposition.

Identify simple propositions,
express the compound proposition in symbolic form,
and determine whether it's true or false.

Answer: The world is flat or zero is an even integer

Let P = "The world is flat" and Q = "zero is an even integer".

Then $P \vee Q$ is "The world is flat or zero is an even integer"

P is false (0), Q is true (1).

According to the definition of $\vee$, $P \vee Q$ is true

(3) The negation of P , denoted $\neg P$, is defined by the truth table

P	$\neg P$
0	1
1	0

Note: The negation of P is "P is false" or any proposition Q equivalent to "P is false" (P and Q cannot be both true and cannot both be false; one is true & the other false)

Ex. " $x > y$ " is NOT the negation of " $x < y$ "
because they could both be false (i.e if $x = y$)

• " $x \geq y$ " is NOT the negation of " $x \leq y$ "
because they could both be true (i.e if $x = y$)

• The negation of " $x < y$ " is ? Ans: $x \geq y$

• The negation of " $x \leq y$ " is ? Ans: $x > y$

— ended here Week 4 Thurs —

To be continued ...

Conditional Statements

Ex from everyday speech

"I'm going to quit if I don't get a raise"

"If it snows, we have a day off"

"I will go to the store provided that my car starts"

All three examples are conditional and can be restated to fit into the form "If CONDITION, then CONCLUSION".

ex: "If my car starts, then I will go to the store"

Def The conditional statement "If P then Q ",

denoted $P \rightarrow Q$, is defined by the truth table

P	Q	$P \rightarrow Q$	
0	0	1	"Case I"
0	1	1	"Case II"
1	0	0	"Case III"
1	1	1	"Case IV"

Ex Suppose I say "If it snows, class is canceled"

Let P be "it snows" and Q be "class is canceled"

Consider the following cases.

Have I told the truth or have I lied?

Case I: P is false and Q is false.
It doesn't snow and class is not canceled.

Ans: I told you the truth

Case II: It doesn't snow and class is canceled.

Ans: I told you the truth

Case III: It snows and class is not canceled. Ans: I lied

Case IV: It snows and class is canceled. Ans: I told you the truth

Other ways to say "If P then Q" in everyday speech and also in technical writing:

"P implies Q"

"Q follows from P"

"Q, if P"

"P is sufficient for Q" ex: "Snowing is a sufficient condition for class to be canceled"

"Q is necessary for P"

Def The converse of the proposition $P \rightarrow Q$ is the proposition $Q \rightarrow P$

Ex The converse of "If it snows, class is canceled" is "If class is canceled, then it snows".

Warning: $Q \rightarrow P$ does NOT have the same logical meaning as $P \rightarrow Q$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

Q is true *P is false*
← *class is canceled, no snow*
← *class is not canceled, snowing*
Q is false *P is true*

Def The contrapositive of the proposition $P \rightarrow Q$ is the proposition $\neg Q \rightarrow \neg P$

Ex The contrapositive of "If it snows, class is canceled" is "If class is NOT canceled, then there is no snow"

Note : What is the connection between $\neg Q \rightarrow \neg P$ and the original proposition $P \rightarrow Q$?

P	Q	$P \rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \rightarrow \neg P$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

$\neg Q \rightarrow \neg P$ has the same logical meaning as $P \rightarrow Q$

Saying

"If it snows, class is canceled"

is logically the same as saying its contrapositive

"If class is NOT canceled, then there is no snow"

Def The inverse of the proposition $P \rightarrow Q$
is the proposition $\neg P \rightarrow \neg Q$

Ex The inverse of "If it snows, class is canceled"
is "If there is no snow, then class is not canceled"

Warning The original proposition and the inverse
are not saying the same thing.

Q What is the connection between the inverse
and the converse of $P \rightarrow Q$?

Def Let P and Q be two propositions. "iff" for short
The biconditional statement " P if and only if Q ",
denoted $P \leftrightarrow Q$, is defined by the truth table

P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

Note $P \leftrightarrow Q$ is true when P and Q have the same truth values.
Otherwise $P \leftrightarrow Q$ is false.

Other ways to say " P if and only if Q " in everyday speech and
also in technical writing:

" P is necessary and sufficient for Q "

" P is equivalent to Q "

"If P , then Q . Also, if Q , then P "