Sec 2.3 Partitions of sets & the Law of Addition

Partitions

Next HW due Tues
Next quiz (Thurs): 1.5, 2.1, 2.2

<u>Def</u> A <u>(set) partition</u> of a set A is a set of one or more nonempty subsets of A:

$A_1, A_2, A_3, \dots$

such that every element of A is in exactly one set.

in symbol: (a) $A_1 \cup A_2 \cup A_3 \dots = A$

(b) If $i \neq j$ then $A_i \cap A_j = \emptyset$ ("pairwise disjoint")

<u>Ex</u> Let A = { Students in this class }

- Then { {1st-year students}, {2nd-year students}, {3rd-year students}, {4-th year students} } is a partition of A
- { {all students in this class} } = {A} is a partition of A
- { {students who came to class today}, {students who are absent} }
- But { {1st-year students}, {2nd-year students}, {3rd-year students}, {4-th year students}, {grad students} } is not a partition of A because there are no grad students in the class.
- { {2nd-year students}, {3rd-year students}, {4-th year students} } is not a partition of A because there are 1st year students in the class.

<u>Ex</u> Let $A = \{1, 2, 3, 4\}$. Partitions of A or not ?

$\{\{1,3\}, \{2,4\}\}$ yes

$\{\{1,2,3\}, \{2,3,4\}\}$ no because the sets are not disjoint

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$\{\{1,2\}, \{3\}, \{4\}\}$ yes

$\{\{1,2\}, \{3\}, \{4\}, \emptyset\}$ No because one of the sets is empty

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$\{\{1,3\}, \{2\}\}$ No because the union of the sets is not A

$\{\{1,3\}, \{2,4,5\}\}$ No because the union of the sets is not A

$\{\{1\}, \{2\}, \{3\}, \{4\}\}$ Yes

$\{\{1,2,3,4\}\}$ yes

$\{1, 2, 3, 4\}$ No, this is a collection of numbers, not a collection of subsets of $\{1,2,3,4\}$.

<u>Ex</u> Consider the set $\mathbb{Z} = \{\text{integers}\}$.

Some examples of partitions of $\mathbb{Z}$:

$$\{\{n\} \mid n \in \mathbb{Z}\} = \{\ldots, \{-1\}, \{0\}, \{1\}, \{2\}, \ldots\}$$

$$\{\underbrace{\{n \in \mathbb{Z} \mid n < 0\}}_{\text{set of negative integers}}, \{0\}, \underbrace{\{n \in \mathbb{Z} \mid n > 0\}}_{\mathbb{P}}\}$$

Addition Law

Thm (Law of addition)

If A is a finite set and if $\{A_1, A_2, \dots, A_n\}$ is a partition of A, then

$$|A| = |A_1| + |A_2| + \dots + |A_n|$$

i.e. If A is a finite set and if $A_1 \cup A_2 \cup \dots \cup A_n = A$ and $\underbrace{A_i \cap A_j = \emptyset \text{ whenever } i \neq j}_{\text{"pairwise disjoint"}}$, then

$$|A| = |A_1 \cup A_2 \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n|.$$

Ex $\{ \overbrace{\{\text{Students who came to class this Tue}\}}^{A_1}, \overbrace{\{\text{Students who were absent this Tue}\}}^{A_2} \}$

is a partition of $A := \{\text{all students in this class}\}$

$$34 = |A| = |A_1| + |A_2| = 29 + 5$$

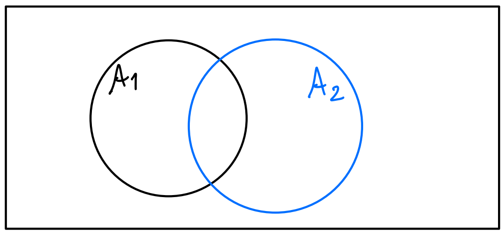
Warning: I cannot add students by majors because some students have multiple majors

Ex A survey 100 people (who owned phones) found that 60 people owned an iphone, 50 people owned Android, and 0 people owned other phones.

How many people had both phones?

Ans Let $A_1 = \{$surveyed people who owned iphone$\}$
$A_2 = \{$ — " — — " — Android$\}$

$A = \{$all people surveyed$\} = A_1 \cup A_2$



$$|A| = |A_1| + |A_2| - \underbrace{|A_1 \cap A_2|}_{\text{to account for the double counting}}$$

$$100 = 60 + 50 - |A_1 \cap A_2|$$

So $\boxed{|A_1 \cap A_2| = 10}$ is the number of people who had both phones.

Thm (Laws of inclusion-exclusion for two sets)
Given finite sets A_1 and A_2,
$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$

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