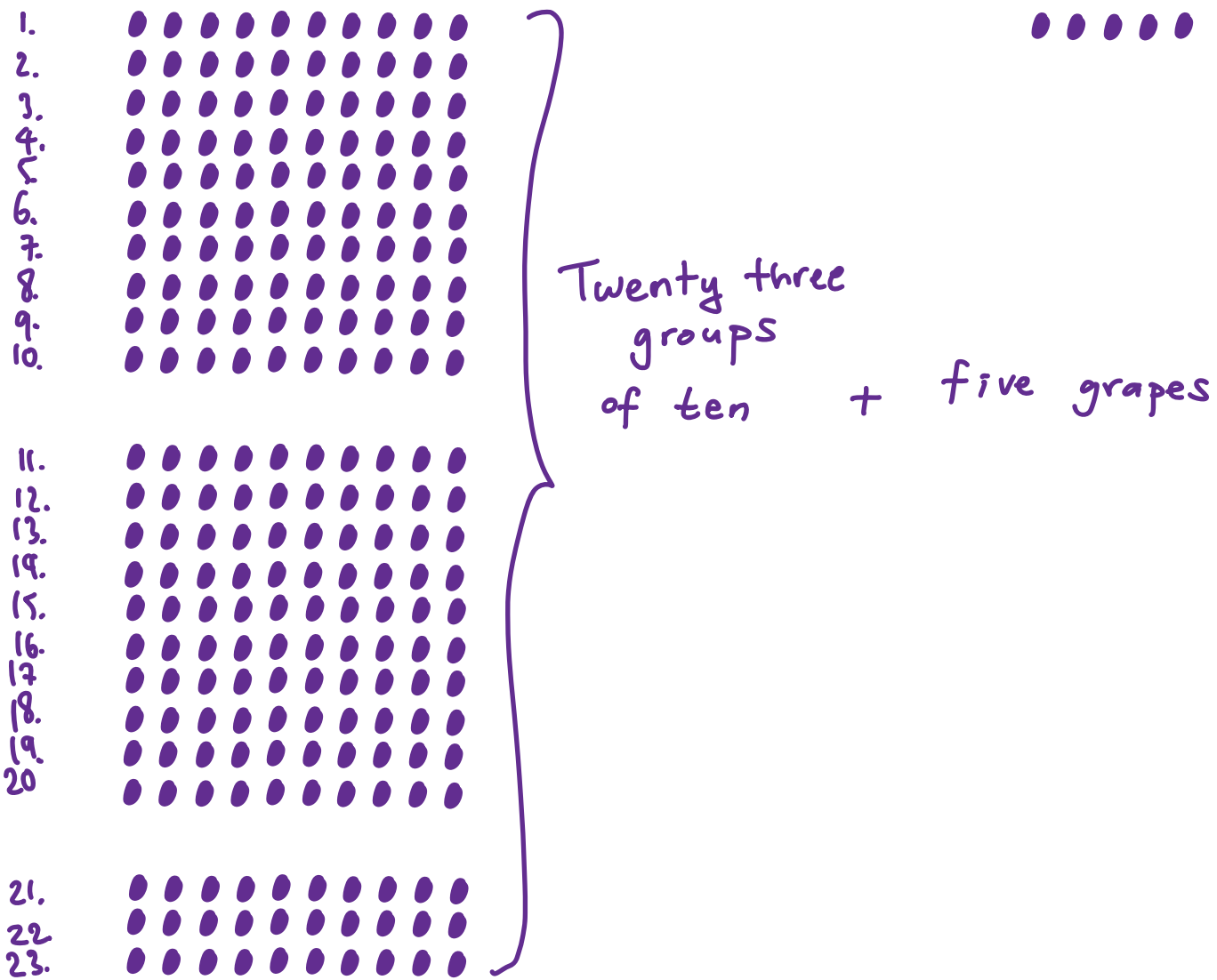
Sec 1.4 Binary representation of positive integers

Counting grapes : group by tens



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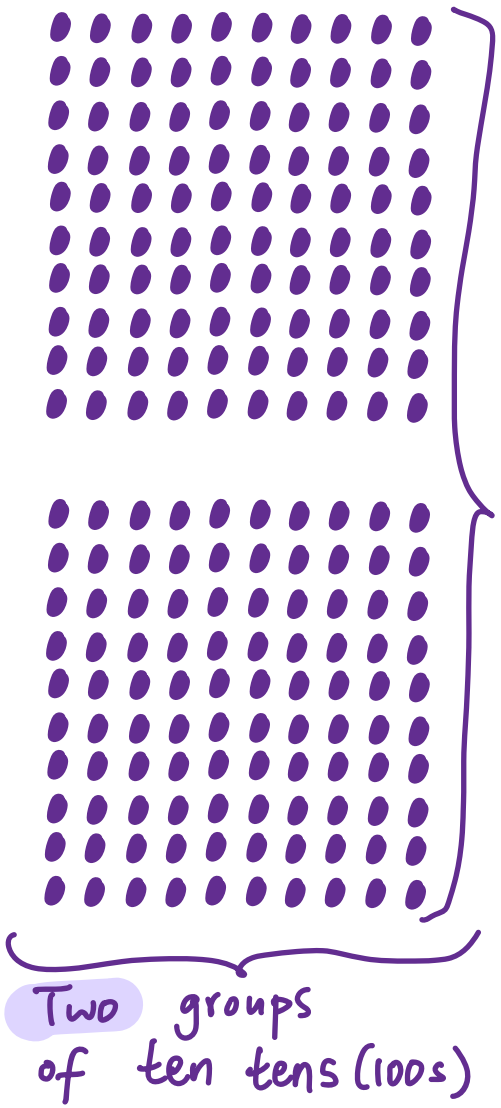
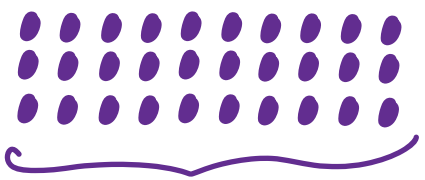
Counting grapes : group by tens t



Two groups of ten tens (100s)



Three groups of ten

five grapes

Record the total as

235

(in decimal)

Find the binary representation of forty one

Set $k = 41$

Divide k by 2 to get

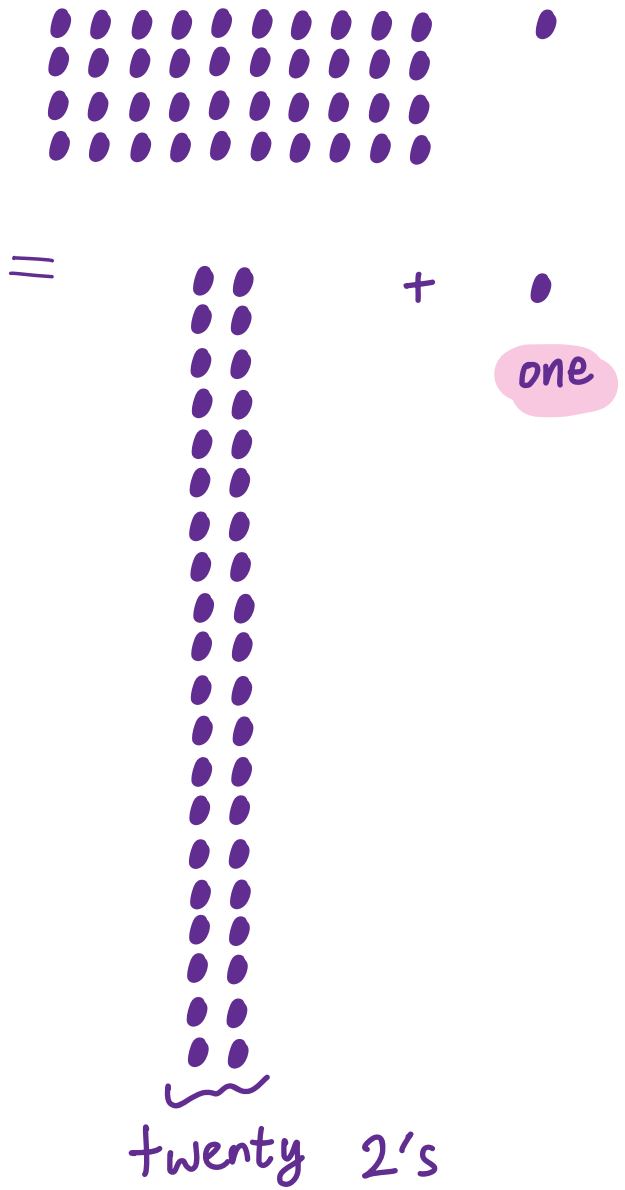
$k = 2 \cdot q + r$

quotient remainder

$41 = 2 \cdot 20 + 1$

Set $k := q$

Now $k = 20$



Now $k = 20$

Divide k by 2 to get

$k = 2 \cdot q + r$

quotient remainder

$20 = 2 \cdot 10 + 0$

Set $k := q$

Now $k = 10$

Now $k = 10$

$k = 2 \cdot q + r$

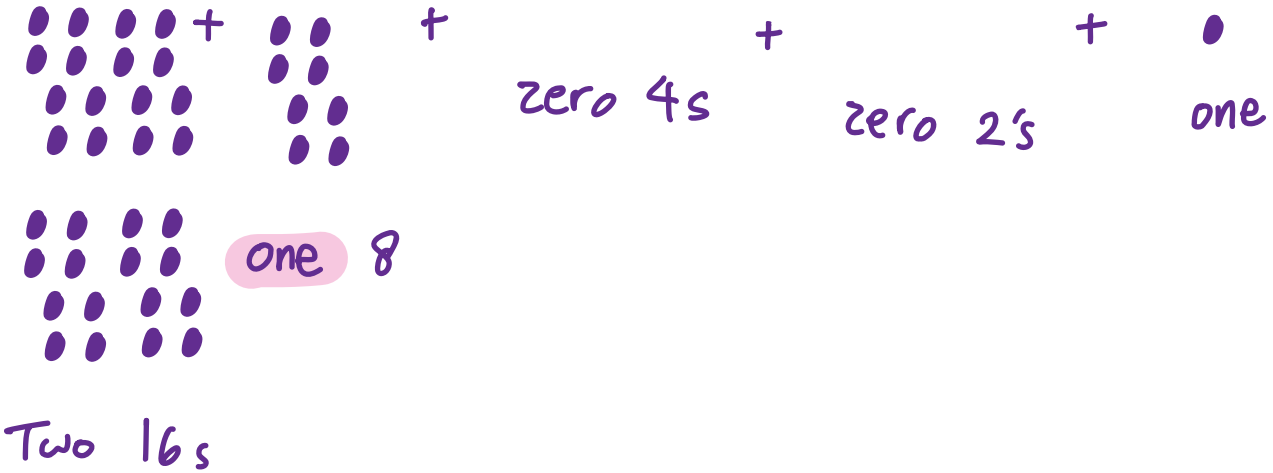
$10 = 2 \cdot 5 + 0$

Set $k := q$

Now $k = 5$

\+ zero 4s + zero 2's + one

Five 8s



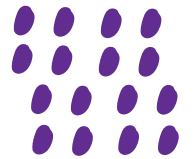
Now $k = 5$

$k = 2 \cdot q + r$

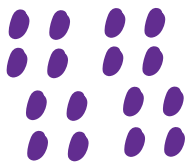
$5 = 2 \cdot 2 + 1$

Set $k := q$

Now $k = 2$


+ zero 16s +

+ zero 4s + zero 2's + one


one 8

One 32

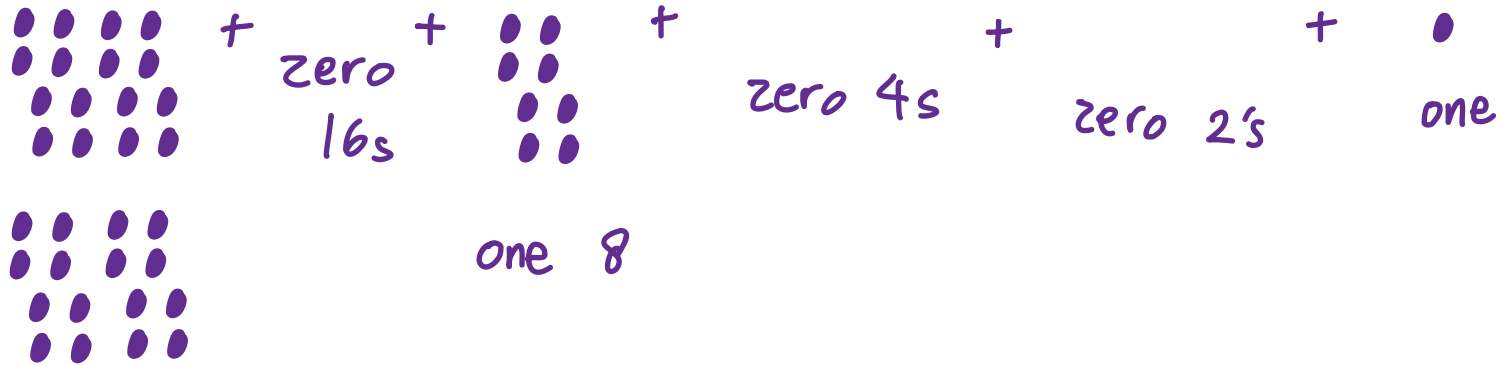
Now $k = 2$

$k = 2 \cdot q + r$

$2 = 2 \cdot 1 + 0$

Set $k := q$

Now $k = 1$



One 32

Now $k = 1$

$k = 2 \cdot q + r$

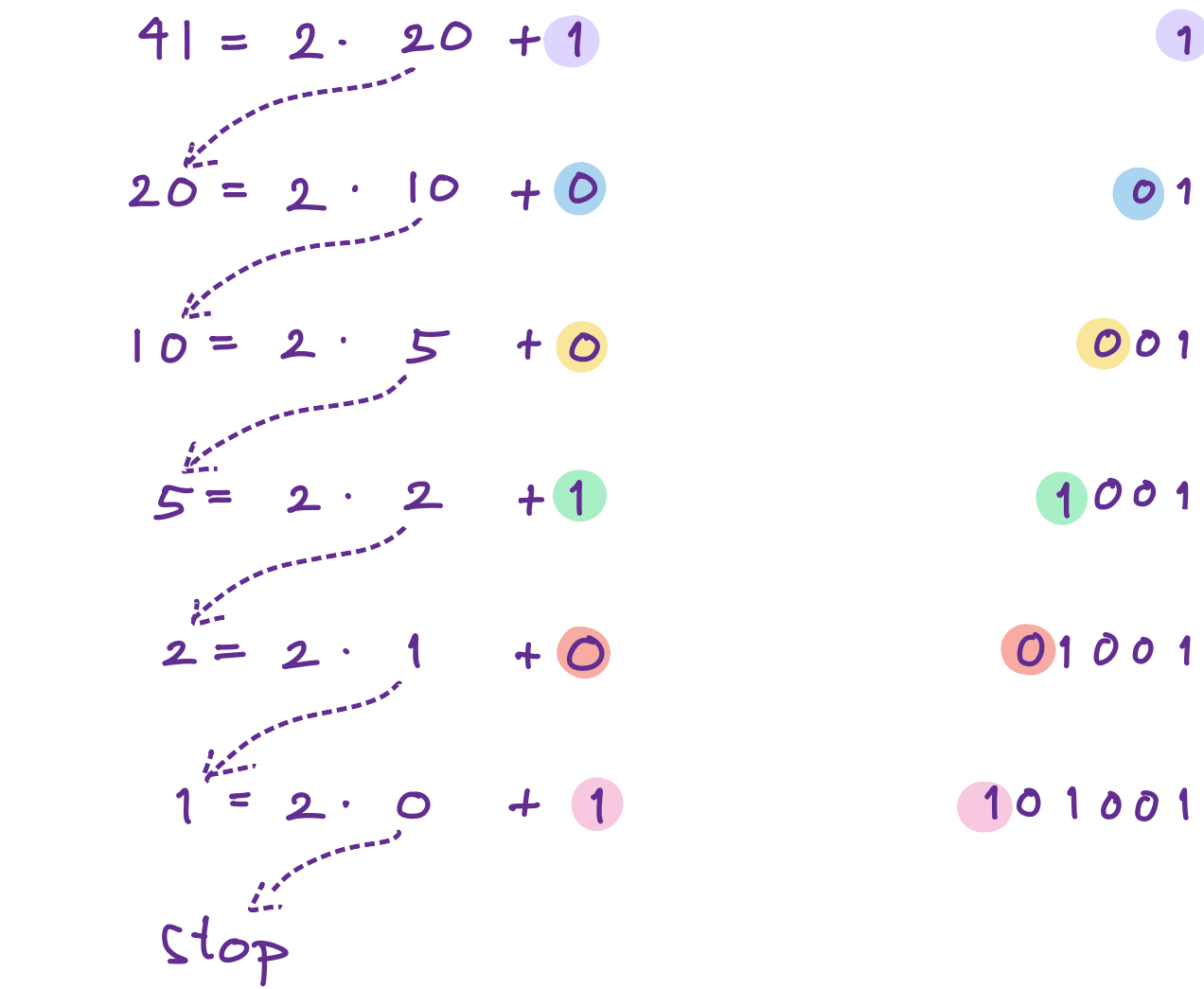
$1 = 2 \cdot 0 + 1$

Set $k := 0$

Terminate the process when $k = 0$

The binary representation of 41 is 101001_{two}
or simply 101001

Computing the binary representation of 41, in one page:



Algorithm to compute the binary representation of any positive integer n:

1. Start with an empty list of bits
2. Set $k := n$
3. While k is positive, do:
   - Divide k by 2 to get $k = 2 \cdot q + r$
   - Attach r to the left of the list of bits (prepend)
   - Set $k := q$

Converting from binary to decimal

Ex 101101_{two} is the binary representation of

$1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 =$

$32 + 8 + 4 + 1 = 45$

Arithmetic in binary

```
  1 1
  1 1 +
-------
1 1 0
```

3 + 3 = 6

```
  1 1
  1 1 x
-------
  1 1
1 1 .
-------
1 0 0 1
```

3 x 3 = 9

```
    1 0 0 1
      1 0 1  x
-------------
    1 0 0 1
      0 .
1 0 0 1 . .
-----------  +
1 0 1 1 0 1
```

9 x 5 = 45

— the end —