Sec 1.2 Basic Set Operations

* No office Hour today after class

* HW 1 due tonight

* HW 2 due Tues

  → You can "resubmit" as many times as you want (prior to deadline)

* Quiz 1 next Thurs

  → 10 mins long, at the end of class

  → Topics: Sec 1.1, 1.2

  → No notes, no electronics

Sec 1.2 Basic Set Operations

Terminology: disjoint sets

<u>Def</u> We say two sets <u>S and T are disjoint</u> if:

S and T have no elements in common.

<u>Ex</u> Let $A = \{ p \mid p \text{ is a prime number}, 2 \le p \le 11 \}$

$B = \{ \text{positive even numbers} \}$

$C = \{ x \mid x \text{ is positive and odd}, \ x \le 11 \}$

B and C are disjoint sets

A and B are not disjoint

A and C are not disjoint

Intersection $\cap$

← looks like the 2nd letter in intersection

Def The <u>intersection</u> of sets S and T (denoted $S \cap T$) is the set of elements that are in both S and T,

$$S \cap T = \{ x \mid x \in S \text{ and } x \in T \}$$

Ex Let $A = \{2, 3, 5, 7, 11\}$

$B = \{2, 4, 6, \ldots\}$

$C = \{1, 3, 5, 7, 9, 11\}$

$A \cap B = \{2\}$

$B \cap C = \varnothing$

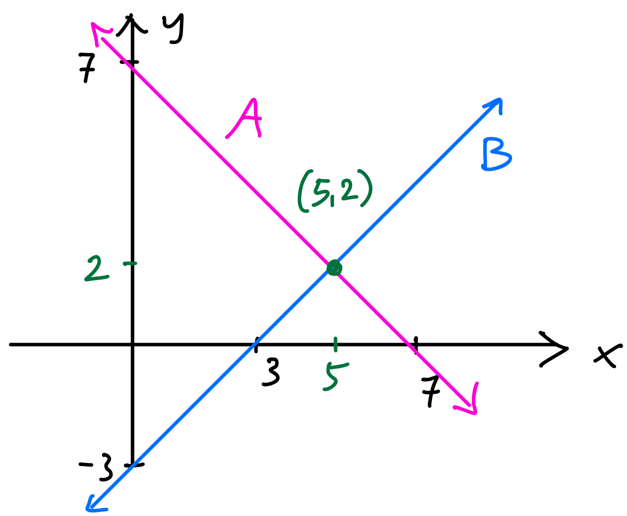
$A \cap C = \{3, 5, 7, 11\}$

Ex $\mathbb{Z} \cap \mathbb{Q} = \mathbb{Z}$

Ex Let $A = \{ (x,y) \mid x + y = 7, \quad x, y \in \mathbb{R} \}$

$= \{$ points on the xy-plane satisfying the equation $x + y = 7 \}$

Let $B = \{ (x,y) \mid x - y = 3, \quad x, y \in \mathbb{R} \}$



Then $A \cap B = \{ (5, 2) \}$

Union

$\cup$ ← looks like the 1st letter in union

Def The union of sets S and T (denoted $S \cup T$) is the set of elements that are in S or in T (or in both S and T)

$$S \cup T = \{x \mid x \in S \text{ or } x \in T\}$$

Ex Let $A = \{2, 3, 5, 7, 11\}$

$C = \{1, 3, 5, 7, 9, 11\}$

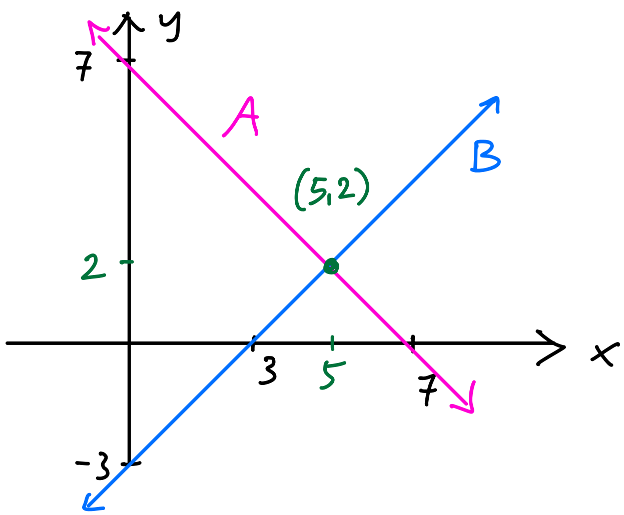
$$A \cup C = \{1, 2, 3, 5, 7, 9, 11\}$$

Ex $\mathbb{Z} \cup \mathbb{Q} = \mathbb{Q}$

Ex $S \cup \emptyset = S$ for any set S

Ex Let $A = \{ (x,y) \mid x+y=7,\ x,y \in \mathbb{R} \}$
$= \{$ points on the xy-plane satisfying the equation $x+y=7 \}$

Let $B = \{ (x,y) \mid x-y=3,\ x,y \in \mathbb{R} \}$



Then $A \cup B = \{$ points on the line A or $B \}$

Is $(5,2) \in A \cup B$ or $(5,2) \notin A \cup B$?

Terminology: disjoint sets

<u>Facts</u> If S and T are disjoint finite sets,
then $|S \cup T| = |S| + |T|$

If S and T are non-disjoint finite sets,
then $|S \cup T| < |S| + |T|$

Universe

Often we need to establish a universe under discussion.

Ex Let the universe be $U = \mathbb{P}$

Then the set $\{x \mid x^2 = 9\}$ is $\{3\}$

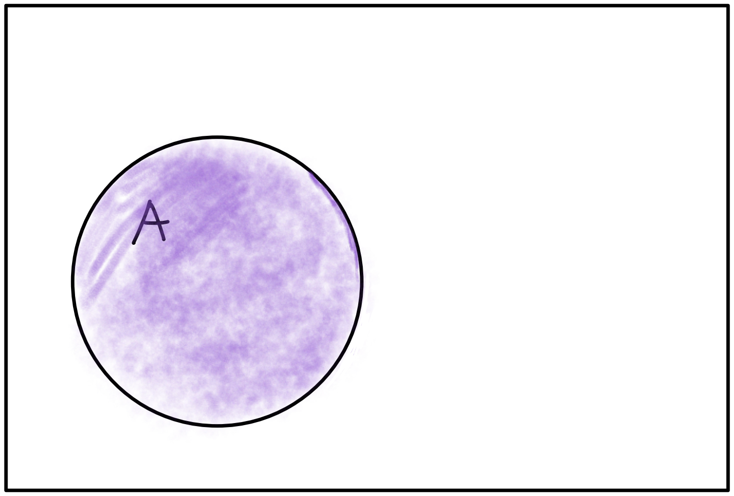
Ex Let the universe be $U = \mathbb{Z}$

Then the set $\{x \mid x^2 = 9\}$ is $\{-3, 3\}$

Venn Diagrams

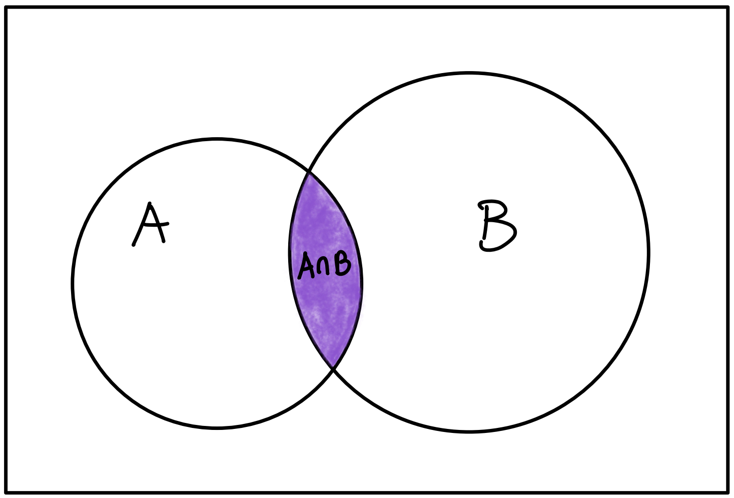
Venn Diagrams

Set A



Intersection

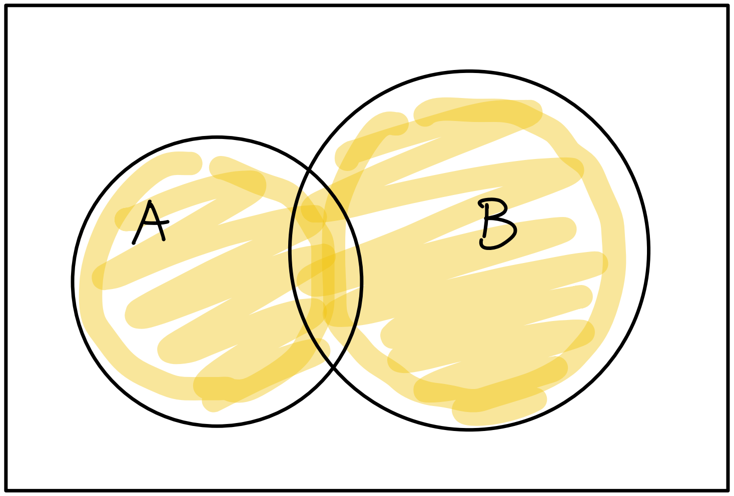
$A \cap B$



Venn Diagrams

Union

$A \cup B$



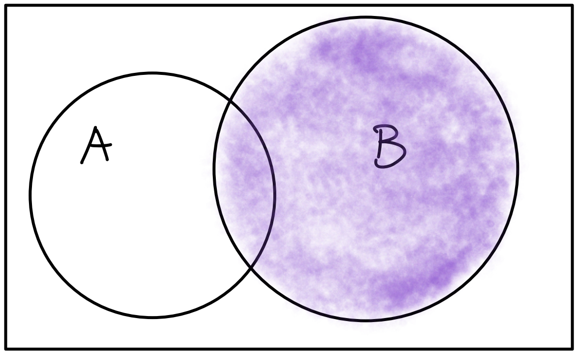
Complement Let A, B be subsets of the universal set U.

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}$$

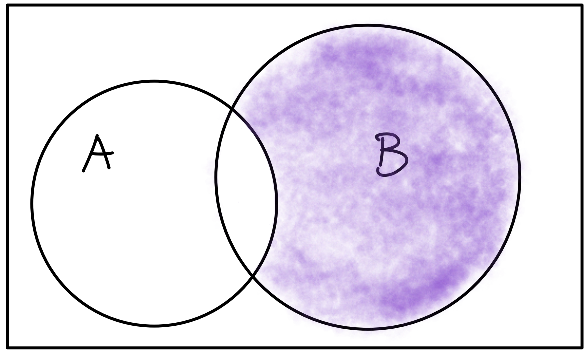
Some books write $B \setminus A$ instead of $B - A$

Read "the complement of B relative to A"

Set B:

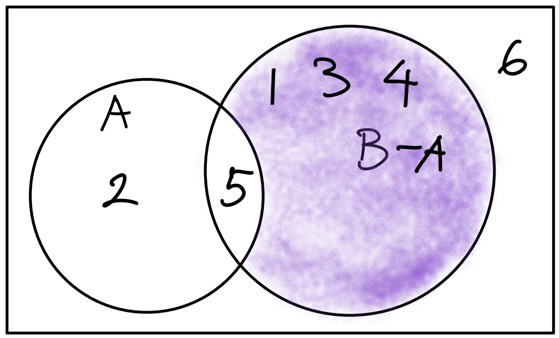


Set $B-A$:



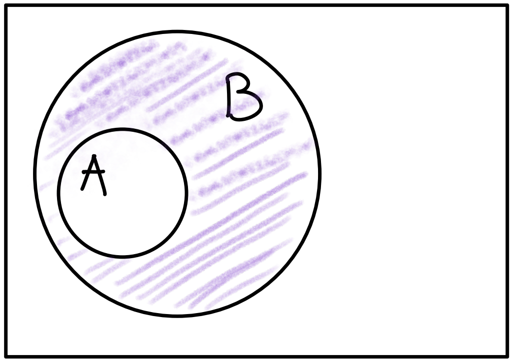
Ex $U = \{1, 2, \ldots, 6\}$, $A = \{2, 5\}$, $B = \{1, 3, 4, 5\}$.

Then $B - A = \{1, 3, 4\}$

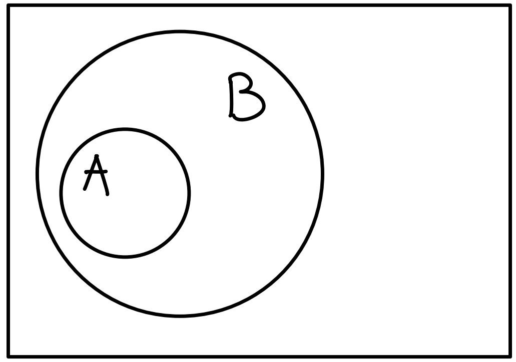


Ex Suppose $A \subset B$

Venn diagram for $B-A$



Venn diagram for $A-B$



In this case, $A-B = \emptyset$

Ex Let $A = \mathbb{N}$, $B = \mathbb{Z}$.

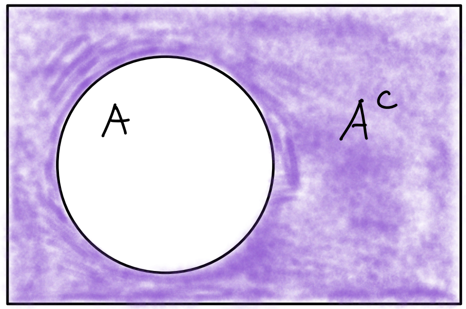
Then $B-A = \{\ldots,-3,-2,-1\}$ and $A-B = \emptyset$

Complement Let A be a subset of the universal set U.

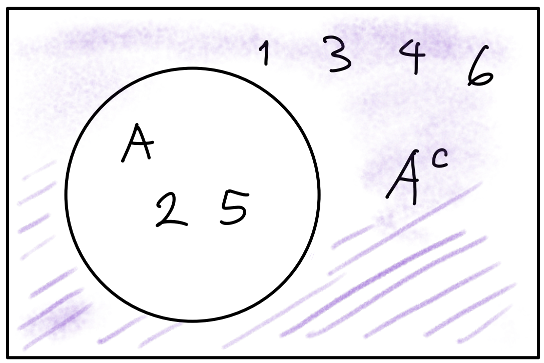
$$A^c = U - A = \{x \in U \mid x \notin A\}$$

(read "the complement of A")

Complement A^c

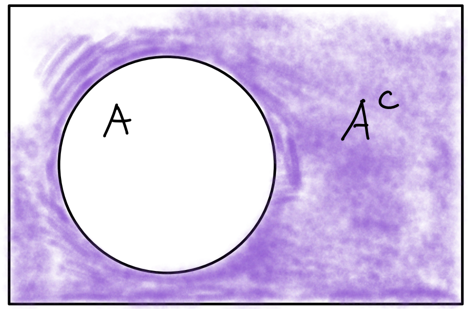


<u>Ex</u> $U = \{1, 2, \ldots, 6\}$, $A = \{2, 5\}$. Then $A^c = \{1, 3, 4, 6\}$



Complement $A^c = U - A = \{x \in U \mid x \notin A\}$

Complement A^c



<u>Ex</u> $U^c = \emptyset$

$\emptyset^c = U$

<u>Ex</u> $U = \mathbb{Z}$. The complement of {even integers} is {odd integers}.

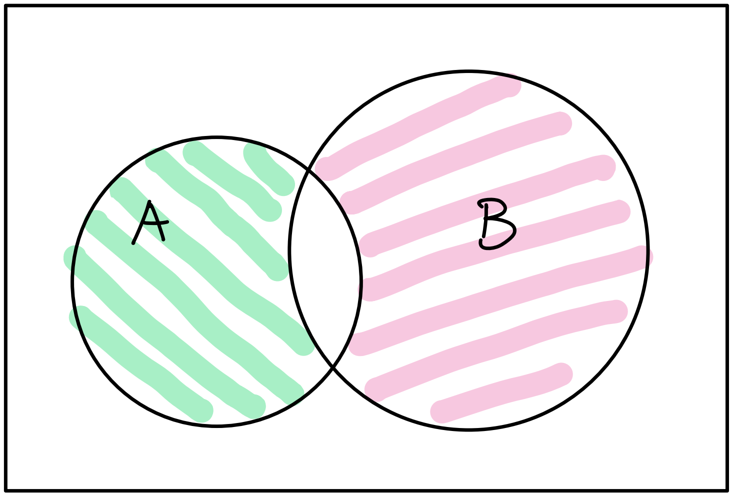
<u>Ex</u> Let $U = \mathbb{R}$. $\mathbb{Q}^c = \{x \in \mathbb{R} \mid x \notin \mathbb{Q}\}$

called <u>irrational numbers</u>

Symmetric difference

$$A \oplus B = (A \cup B) - (A \cap B) \quad ,$$

Read "the symmetric difference of A and B"

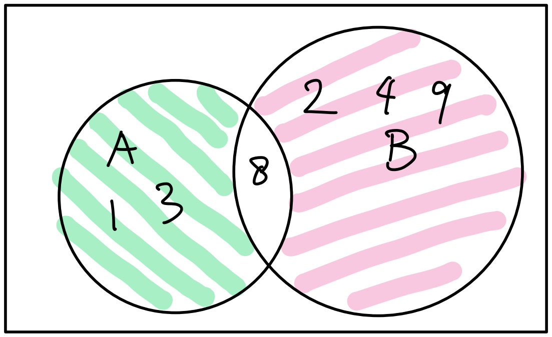


<u>Ex</u> $\left.\begin{array}{l} A \oplus \varnothing = A \\ A \oplus A = \varnothing \end{array}\right\}$ for any set A

Symmetric difference

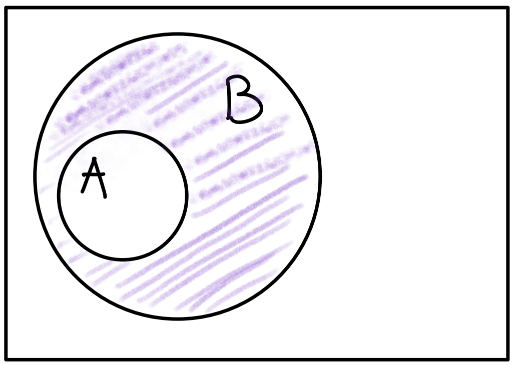
$$A \oplus B = (A \cup B) - (A \cap B)$$

Ex Let $A = \{1, 3, 8\}$, $B = \{2, 4, 8, 9\}$. Then $A \oplus B = \{1, 2, 3, 4, 9\}$



Ex If $A \subseteq B$ then $A \oplus B = B - A$

Ex : $\mathbb{Q} \oplus \mathbb{R} = \mathbb{R} - \mathbb{Q} = \{\text{irrational numbers}\}$



Exercise

Suppose U is an infinite universal set, and A and B are infinite subsets of U.

1) Must A^c be finite?
   Must A^c be infinite?

   Both are possible.

   Find examples U and A for finite / infinite

2) Must $A \cup B$ be finite / infinite?

   Must be infinite. Why?

3) Must $A \cap B$ be finite / infinite? Both are possible.

   Find examples U, A, B for finite / infinite

4) Must $A \oplus B$ be finite / infinite? Both are possible.

   Find examples U, A, B for finite / infinite

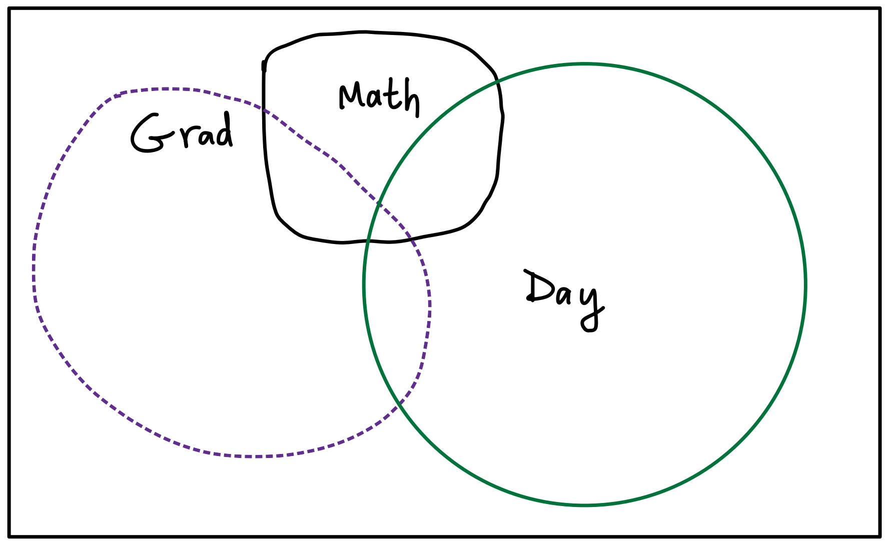
Exercise #7

Let $U = \{\text{all students at UML}\}$,

$D = \{\text{day students}\}$,

$M = \{\text{math majors}\}$,

$G = \{\text{grad students}\}$



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Let $U = \{\text{all students at UML}\}$,

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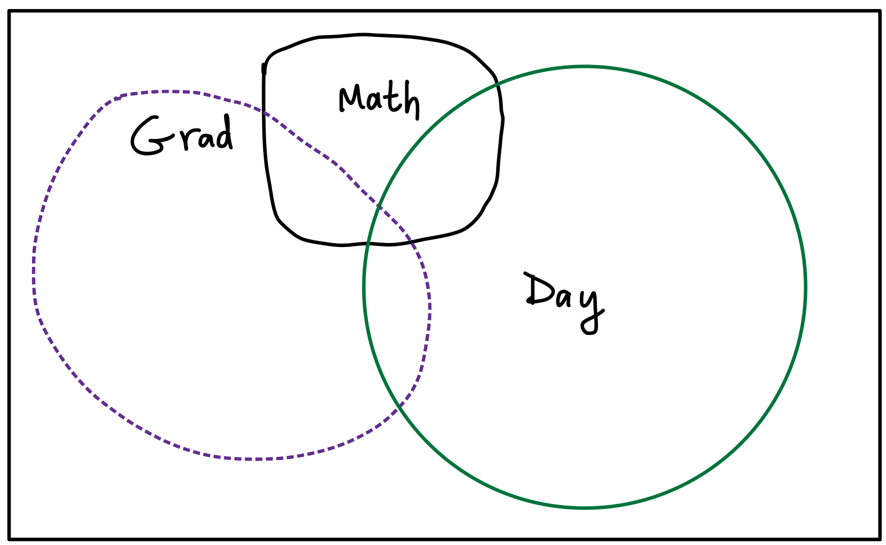
Venn diagram for $\{\underbrace{\text{evening students}}_{D^c = U - D}\}$



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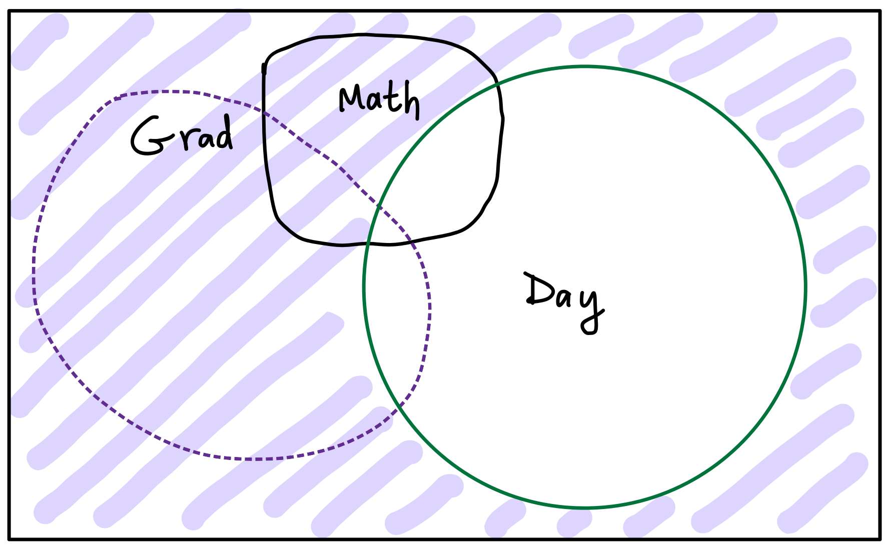

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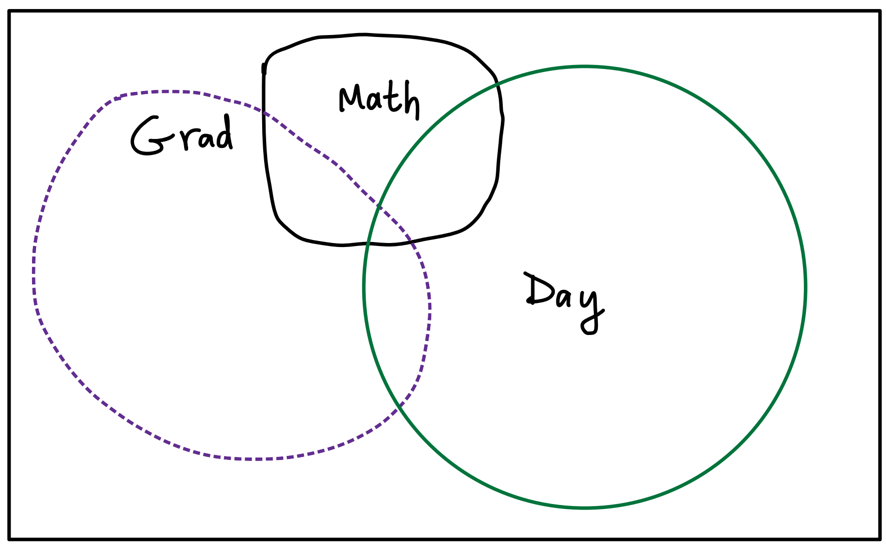
Venn diagram for $\{\underbrace{\text{undergraduate}}_{G^c} \underbrace{}_{\text{and}} \underbrace{\text{math majors}}_{M}\}$



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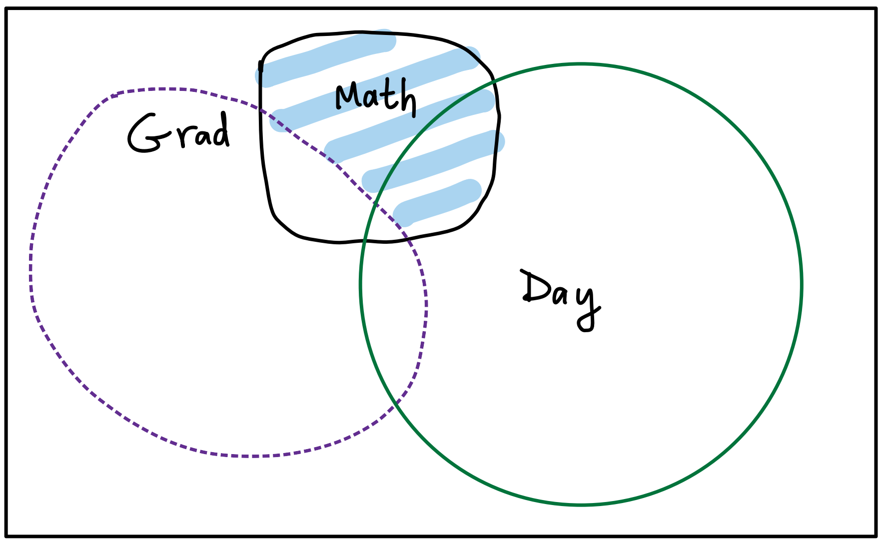

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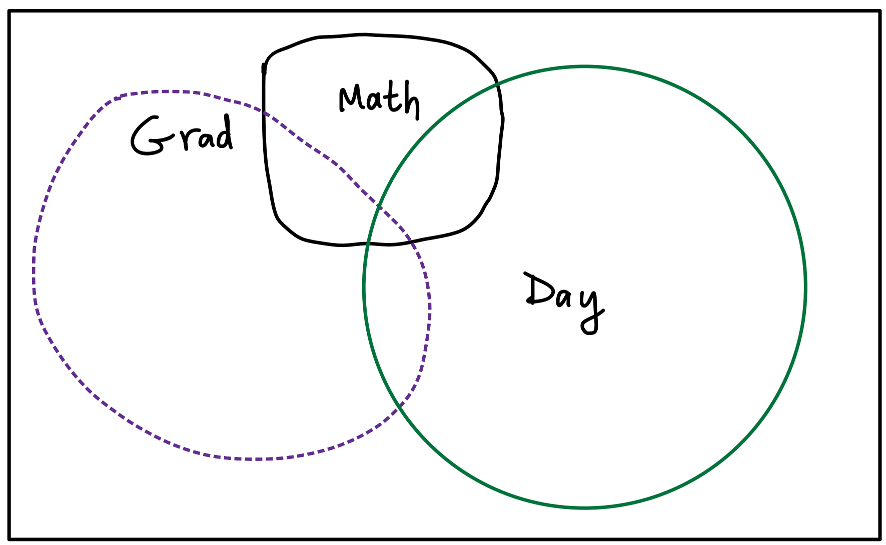
Venn diagram for $\{\underbrace{\text{undergraduate}}_{G^c}\ \text{and}\ \text{non-}\underbrace{\text{math majors}}_{M^c}\}$



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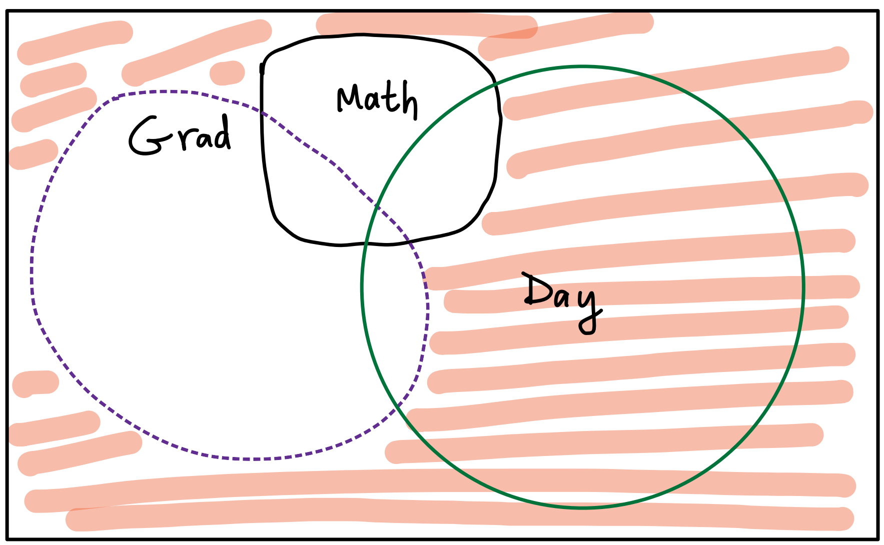
Venn diagram for $\{\underbrace{\text{undergraduate}}_{G^c}\ \text{and}\ \underbrace{\text{non-math majors}}_{M^c}\}$



Note:

$G^c \cap M^c = (G \cup M)^c$

1.2.3 SageMath Note: Sets

To work with sets in Sage, a set is an expression of the form Set(*list*). By wrapping a list with `Set( )`, the order of elements appearing in the list and their duplication are ignored. For example, L1 and L2 are two different lists, but notice how as sets they are considered equal:

```
L1=[3,6,9,0,3]
L2=[9,6,3,0,9]
[L1==L2, Set(L1)==Set(L2) ]
```

Evaluate (Sage)

The standard set operations are all methods and/or functions that can act on Sage sets. *You need to evaluate the following cell to use the subsequent cell.*

```
A=Set(srange(5,50,5))
B=Set(srange(6,50,6))
[A,B]
```

Evaluate (Sage)

We can test membership, asking whether 10 is in each of the sets:

```
[10 in A, 10 in B]
```

Evaluate (Sage)

The ampersand is used for the intersection of sets. Change it to the vertical bar, |, for union.

```
A & B
```

Evaluate (Sage)

Symmetric difference and set complement are defined as "methods" in Sage. Here is how to compute the symmetric difference of A with B, followed by their differences.

```
[A.symmetric_difference(B),A.difference(B),B.difference(A
```

← The end of textbook Sec 1.2

SageMath.org

Sec 1.1 Exercise #8 (Extra)

One reason that we left the definition of a set vague is Russell's Paradox:

- The "Russell set" R is the set of all sets that are not elements of themselves

$$R = \{ x \mid x \notin x \}$$

- If $R \in R$, then by definition $R \notin R$.
- If $R \notin R$, then by definition $R \in R$.

Barber Paradox (Extra)

- Suppose there is a barber B
- B shaves all people (and only those) who do not shave themselves.
- If B shaves himself, then B is not in All people who do not shave themselves

  But B only shaves people in All people who do not shave themselves

- If B does not shave himself, then B is in All people who do not shave themselves

  But B shaves all people in All people who do not shave themselves .

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HW 2 due Tues

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→ No office Hour today after class

— end —