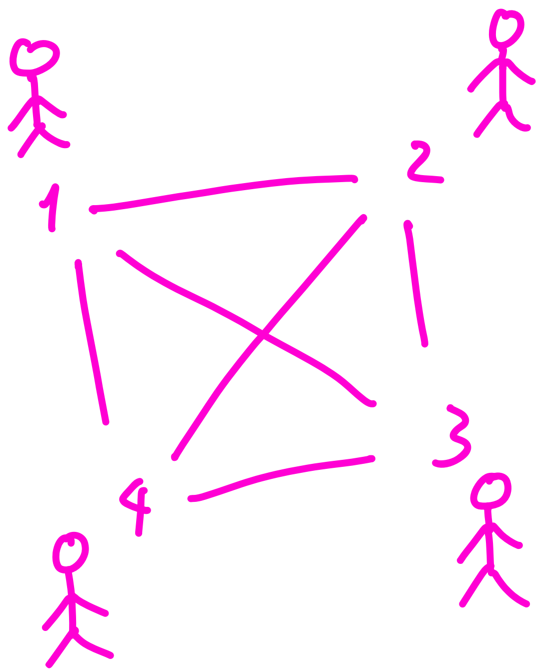
Math 2190 Discrete Structures I

* Form sets of cardinality 3 or 4.
    - If your group is close to a board, please claim a section of the board
* Introduce yourself to your set members
* Share a fun fact or favorite movie/music/book/color
* Share a current hobby (or a past/future hobby)

Within your group, shake everybody's hand exactly once.

* How many handshakes happened?

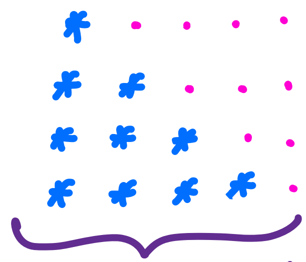


For a group of size 4, there are six handshakes

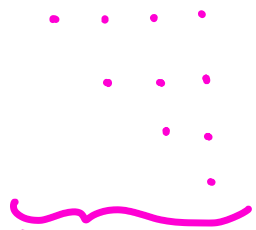
Within your group, shake everybody's hand exactly once.

* If there are 5 people in your group, how many handshakes would occur?

Total 10 handshakes



5.4 total



$\frac{5 \cdot 4}{2} = 10$ dots

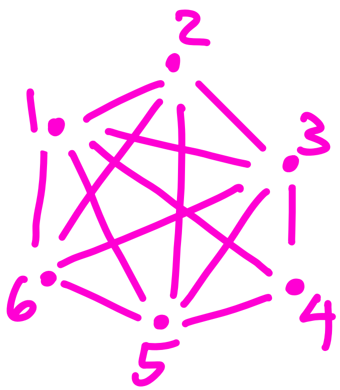
$$\overbrace{4+\overbrace{3+2}^{5}+1}^{5} = \underbrace{(4+1)} + \underbrace{(3+2)} =$$

$$= \left(\frac{5}{2}+\frac{5}{2}\right)+\left(\frac{5}{2}+\frac{5}{2}\right)$$

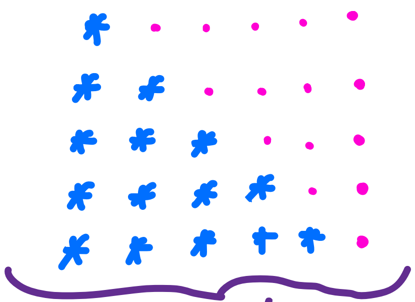
$$= \left(\frac{5}{2}\right)4$$

Within your group, shake everybody's hand exactly once.

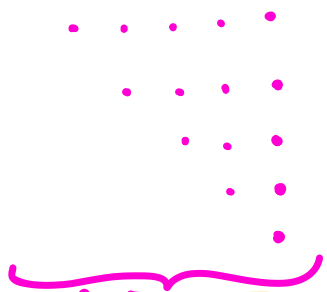
* If there are 6 people in your group, how many handshakes would occur?



Total 15 handshakes



$6 \cdot 5$ total



$\frac{6 \cdot 5}{2} = 15$ dots

$$5+4+3+2+1$$

$$= (5+1) + (4+2) + 3$$

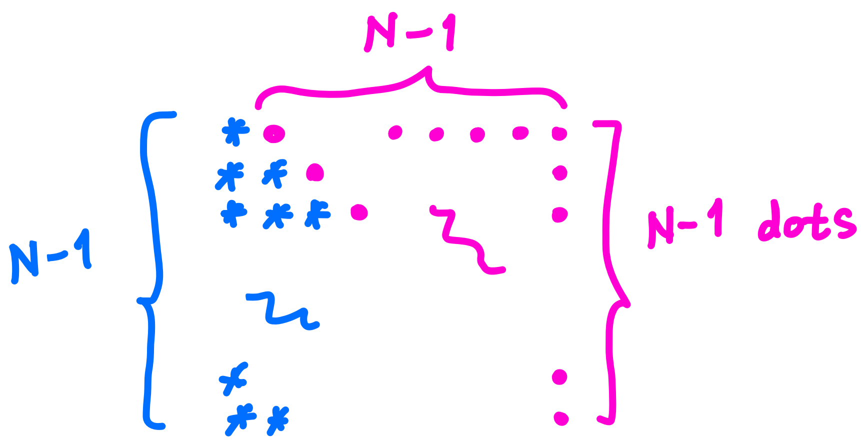
$$= \left(\frac{6}{2}+\frac{6}{2}\right) + \left(\frac{6}{2}+\frac{6}{2}\right) + \frac{6}{2}$$

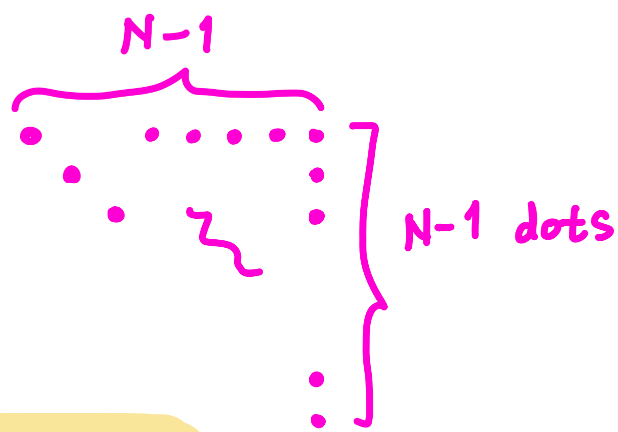
$$= \left(\frac{6}{2}\right)5$$

Within your group, shake everybody's hand exactly once.

* If there are N people in your group, how many handshakes would occur?

$$\sum_{k=1}^{N-1} k = (N-1) + (N-2) + (N-3) + \cdots + 3 + 2 + 1$$





$N(N-1)$ total

$\boxed{\dfrac{N(N-1)}{2}}$ total

Within your group, shake everybody's hand exactly once.

* If there are 5 people in your group, how many handshakes would occur?

- Counting the number of handshakes is the same as counting a pair of people.
- First, choose a person A (5 choices).
- Choose a 2nd person B (4 choices).
- There are $5 \cdot 4 = 20$ possibilities.
- But choosing B first then A would result in the same pairing.
- So there are $\frac{5 \cdot 4}{2} = 10$ possible pairs.

* If there are N people in your group, how many handshakes would occur?

$N(N-1)/2$

Math 2190 Discrete Structures I

Instructor: Emily Gunawan (Southwick 350M)

→ Call me either "Dr. Gunawan" or "Emily"

Email: emily_gunawan@uml.edu

Course webpage: canvas.uml.edu

→ See syllabus

Class meetings: Tue & Thu 3:30 - 4:45pm @Olsen 405

Class recordings are on Canvas

Math 2190 Discrete Structures I

Office Hours (Drop in any time):

(Only during first eight weeks)
Prof. Yankowskas
Southwick 301 B

- Mon 12–1:30 pm
- Fri 12–1:30 pm

Prof. Gunawan
Southwick 350 M

- Tue 2-3pm, 5–5:30pm
- Thurs 2-3pm, 5–5:30pm

Tutoring services for Discrete I:

M–F 10am–5pm @ Southwick 310

Sat, Sun 5–8pm via Zoom

CLASS tutoring, various locations

Math 2190 Discrete Structures I

* Keep a notebook to record your work for suggested exercises & reading

10% Weekly HW (Canvas → Gradescope)

HW 1: this Thurs ("survey")

HW 2: next Tue (Sec 1.1 & 1.2)

20% Weekly short quizzes —— no sheet

Quiz 1 next Thurs (Sec 1.1 and 1.2)

20% Exam 1 (Thurs Oct 1)

20% Exam 2 (Thurs Nov 5)

30% Final Exam (TBD by Registrar) during 12/12 – 12/19

hand-written sheet allowed on exams

→ Calculators not needed

Math 2190 Discrete Structures I

* Discrete Math is related to most of my research :)

* This course caters to CS majors & math majors, but many of the topics we'll cover should be of interest to students majoring in other sciences.

* Questions?

Sec 1.1 Set notation & relations

A <u>set</u> is a collection of objects
called "elements of a set"

$x \in S$ means "x is an element of the set S"
or $x \,\varepsilon\, S$

$x \notin S$ means "x is not an element of the set S"

<u>Ex</u> $B = \{0, 1\}$ is the set of binary digits

"$0 \in B$" is a true statement

"$1 \in B$" — " —

"$0, 1 \in B$" — " —

"$2 \in B$" is a false statement

"$B = 0$" is a false statement

"$B = 1$" — " —

A compact way to write "2 is not an element of B"
is "$2 \notin B$"

on the board I might
write "elt" for short

must use curly brackets — square brackets [] and parentheses () have different meanings

Ex $\{2\}$ is the set containing just the number 2.

It is a set, not a number.

Analogy: a purse containing just one coin is not the same thing as a coin.

Ex The empty set is the set containing no elements at all.

Notation: $\{\}$ or $\varnothing$

Analogy: an empty purse

Ex What are the differences between these, if any?

(a) $\{0\}$ set containg one element

(b) $\varnothing$ set (empty)

(c) 0 not a set

We can use ellipsis $(\ldots)$ to list many elements or infinitely many elements

<u>Ex</u> The letters of the alphabet can be described as $A = \{a, b, c, \ldots, x, y, z\}$

<u>Ex</u> The integers from 1 to 1000 could be described as $G = \{1, 2, \ldots, 999, 1000\}$

Two sets S and T are <u>equal</u> if:

every element of S is an element of T and

every element of T is an element of S

<u>Ex</u> These are all the same set:

- $\{4, 5\}$
- $\{5, 4\}$ order doesn't matter!
- $\{4, 5, 5\}$ Repetition doesn't matter!

We will drop the repetition

<u>Ex</u>

- $\{\{1,2\}, \{3,4\}\}$ has two elements, $\{1,2\}$ and $\{3,4\}$
- Is $\{\{1,2\}, \{3,4\}\}$ the same as $\{\{1,3\}, \{2,4\}\}$?

Ex Are the sets

$$\{\{1,2\},\{3,4\}\} \text{ and } \{\{4,3\},\{2,1\}\}$$

the same or different?

Ex Are the sets

$$\{\{1,2,3\},\{4,5,6\}\} \text{ and } \{\{1,2\},\{3,4\},\{5,6\}\}$$

the same or different?

Standard symbols

What is $\mathbb{N}$? The natural numbers
$\{0, 1, 2, 3, \ldots\}$

What is $\mathbb{P}$? The positive integers
$\{1, 2, 3, \ldots\}$

What is $\mathbb{Z}$? Set of integers $\{\ldots, -1, 0, 1, 2, \ldots\}$

What is $\mathbb{Q}$? The rational numbers (see next page)

What is $\mathbb{R}$? The "real numbers"
(Think of $\mathbb{R}$ as the set of points on a number line)

Set-builder notation

<u>Ex</u> The set of rational numbers $\mathbb{Q}$ can be defined as

indicates a typical element of the set is a "fraction"

$$\left\{ \overbrace{\frac{a}{b}} \;\middle|\; \underbrace{a,b \in \mathbb{Z}}, \; b \neq 0 \right\} \qquad \text{or}$$

short for "a and b are integers"

read comma as "and"

$$\left\{ \frac{a}{b} : a,b \in \mathbb{Z}, \; b \neq 0 \right\}$$

read "where" or "such that"

Read "the set of fractions a/b such that a and b are integers and b is not equal to zero"

Ex Let $S = \{ x \in \mathbb{R} \mid 2x \in \mathbb{P} \}$

set of positive integers

Which is true?

(A) $S = \{2, 4, 6, 8, \dots\}$

(B) $S = \{0, 2, 4, 6, 8, \dots\}$

(C) $S = \{1/2, 1, 3/2, 2, \dots\}$

(D) $S = \{0, 1/2, 1, 3/2, 2, \dots\}$

1 minute

Read "the set of all real numbers x such that $2x$ is a positive integer"

Ex Let $S = \{ x \in \mathbb{R} \mid 2x \in \mathbb{P} \}$

Which is true?

Answer:

(c) $S = \{ 1/2, 1, 3/2, 2, \ldots \}$

Note: $T = \{ 2x \in \mathbb{R} \mid x \in \mathbb{P} \}$ is $\{ 2, 4, 6, \ldots \}$

Ex Express the set of all odd positive integers in set-builder notation

Possible answers

$$\{2x+1 \mid x \in \mathbb{N}\}$$

$$\{2k-1 \mid k \in \mathbb{P}\}$$

$$\{2j+1 \mid j \in \mathbb{Z},\ j \geq 0\}$$

<u>Note</u>

- Notation is meant to be a help, not a hindrance.

  Brevity of notation is <u>not</u> the goal.

- There are usually many different (equally good) ways of describing a set

<u>Ex</u> $\{x \in \mathbb{R} \mid x^4 = 1\} = \{\ ?\ \}$

$\{x \mid x \in \mathbb{R}, x^4 = 1\} = \{\ ?\ \}$

Note. • Notation is meant to be a help, not a hindrance.

Brevity of notation is not the goal.

• There are usually many different (equally good) ways of describing a set

Ex $\{x \in \mathbb{R} \mid x^4 = 1\}$ and $\{x \mid x \in \mathbb{R}, x^4 = 1\}$

both describe $\{-1, 1\}$

Cardinality

Def. • A set is <u>finite</u> if it has a finite number of elements

• A set which is not finite is called an <u>infinite</u> set

Def The <u>cardinality</u> of a finite set S, denoted $|S|$, is the number of different elements in S.

Ex

$|\{4,5\}| = 2$

$|\{4,5,5\}| = 2$ Drop the repetition

$|\emptyset| = 0$

$|\{0\}| = 1$

Subsets

Def Let S and T be sets. We say

"S is a subset of T"

if:

Every element of S is an element of T.

Notation "$S \subseteq T$" ("$S \subset T$" in some books) is a compact way to denote the fact that S is a subset of T

Ex $\mathbb{P} \subseteq \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

Note
- If $S \subseteq T$ and $T \subseteq S$ then $S = T$.
- If $S = T$, then $S \subseteq T$ and $T \subseteq S$.

What is the difference between these two symbols?

$$\in \quad \text{and} \quad \subseteq$$

Note $S \subseteq S$ and $\emptyset \subseteq S$

Def We say "S is a proper subset of T"

(notation: $S \subsetneqq T$ or $S \subset T$ in some books)

if S is a subset of T and S is not T

(notation: $S \subseteq T$ and $S \neq T$)

Ex $\mathbb{P} \subsetneqq \mathbb{N} \subsetneqq \mathbb{Z} \subsetneqq \mathbb{Q} \subsetneqq \mathbb{R}$

Theorem Suppose S and T are finite sets.

* If $S = T$, then $|S| = |T|$
* If $S \subseteq T$, then $|S| \leq |T|$
* If $S \subsetneqq T$, then $|S| < |T|$

Sec 1.1 Exercise #5 (b)

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, and $C = \{1, 5, 9\}$.

True or false? $\boxed{\{3\} \in A}$

Why?

Sec 1.1 Exercise #5 (c)

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, and $C = \{1, 5, 9\}$.

True or false? $\boxed{\{3\} \subseteq A}$

Why?

Sec 1.1 Exercise #5 (g)

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, and $C = \{1, 5, 9\}$.

True or false? $\boxed{\emptyset \in A}$

Why?

Sec 1.1 Exercise #5 (g')

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, and $C = \{1, 5, 9\}$.

True or false? $\boxed{\emptyset \subseteq A}$

Why?

Sec 1.1 Exercise #6

Is there a set A which satisfies $A = \{2, |A|\}$?

If not, explain why not.

See recording

Sec 1.1 Exercise #8 (Extra)

One reason that we left the definition of a set vague is Russel's Paradox:

The "Russell set" R is the set of all sets that are not elements of themselves

$$R = \{ x \mid x \notin x \}$$

If $R \in R$, then by definition $R \notin R$.

If $R \notin R$, then by definition $R \in R$.

To do:

→ Submit HW 1 (Canvas > HW > Gradescope)
due this Thursday

→ Read Textbook Sec 1.1 and 1.2
+ try the exercises

— end —