Math 2220 Linear Algebra Ch 1 Review

1 Section 1.1

Consider the system (of linear equations)

$$\begin{array}{ccccl} x_1 & -2x_2 & -x_3 & +3x_4 & = 1 \\ 2x_1 & -4x_2 & +x_3 & & = 5 \\ x_1 & -2x_2 & +2x_3 & -3x_4 & = 4 \end{array} \tag{1.1}$$

2 Sec 1.3 Vector equations

Write an vector equation that is equivalent to the system (1.1)

3 Sec 1.4 The matrix equation $A\mathbf{x} = \mathbf{b}$

Write a matrix equation that is equivalent to the system (1.1)

Let $A = \begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix}$.

4 Sec 1.1 Systems of linear equations and Sec 1.2 Row reduction and echelon forms

Is the system from (1.1), $\left\{\begin{array}{ccccl} x_1 & -2x_2 & -x_3 & +3x_4 & = 1 \\ 2x_1 & -4x_2 & +x_3 & & = 5 \\ x_1 & -2x_2 & +2x_3 & -3x_4 & = 4 \end{array}\right.$ consistent or inconsistent?

Fact: A system of linear equations has ...

1. no solutions, or
2.
3.

(Con't answering Question 4)

The augmented matrix of the system (1.1) is

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 2 & -4 & 1 & 0 & 5 \\ 1 & -2 & 2 & -3 & 4 \end{array}\right] \tag{4.1}$$

Row reduce:

An echelon form of the augmented matrix (4.1) gives us enough information to determine whether a system is consistent/inconsistent because of the following theorem.

> Theorem:
> A linear system is ________________ if and only if

Since an echelon form of the augmented matrix (4.1) ____________________________, the system (1.1) is ________________.

Optional: Continue to row reduce to obtain the UNIQUE ____________________________________:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

5 Sec 1.3 and 1.4

Is $\begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$ in the subset of $\mathbb{R}^3$ spanned by the columns of $A = \begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix}$? Why/why not?

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$.
The subset of $\mathbb{R}^3$ spanned (or generated) by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$, denoted

________________________________,

is ...

6 Sec 1.5 Solution sets of linear systems

(a) Describe all solutions of the ______________________ system

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 2 & -4 & 1 & 0 & 5 \\ 1 & -2 & 2 & -3 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The variables that don't correspond to the pivot columns, x_2 and x_4, are called ______________________.

The solutions are ...

(b) Describe all solutions of the ______________________ system

$$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & -4 & 1 & 0 \\ 1 & -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & -1 & 3 & 1 \\ 2 & -4 & 1 & 0 & 5 \\ 1 & -2 & 2 & -3 & 4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

The solutions are ...

(c) Does $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

Fact:

(a) A homogeneous linear system has at least one solution $\mathbf{x}$=$\mathbf{0}$, called the __________________________.

(b) A _______________________ is a __________________________ satisfying $A\mathbf{x} = \mathbf{0}$.

7 Sec 1.7 Linear independence

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, and $\mathbf{v}_4 = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$.

(a) Determine (using work we've already done) whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

Fact:
The columns of a matrix M are linearly independent if and only if the matrix equation $M\mathbf{x} = \mathbf{0}$ has ____.

(b) Determine by inspection whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.