Recommended Problems Section 9.3

#1 Evaluate $\int_2^\infty \frac{dx}{(5x-2)^6}$

#2 Determine whether $\sum_{n=2}^{\infty} \frac{1}{(5n-2)^6}$ converges or diverges

#3 (i) Evaluate $\int_1^\infty x^2 e^{-x^3}\, dx$

(ii) Determine whether $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges

#4 (i) Evaluate $\int_2^\infty \frac{1}{x[\ln(4x)]^2}\, dx$

(ii) Determine whether $\sum_{n=2}^{\infty} \frac{1}{n} \frac{1}{(\ln(4n))^2}$ converges or diverges

Recommended Problems Sec 9.3 Solutions

#1 Evaluate $\displaystyle\int_2^{\infty} \frac{dx}{(5x-2)^6}$

Sol:

$$\int_2^t \frac{1}{(5x-2)^6}\,dx = \int_{u=5(2)-2}^{u=5t-2} u^{-6}\,\frac{1}{5}\,du = \frac{1}{5}\,\frac{u^{-5}}{-5}\Bigg|_{u=8}^{u=5t-2}$$

$u = 5x-2$

$du = 5\,dx$

$\frac{1}{5}\,du = dx$

$$= -\frac{1}{25}\left[\frac{1}{u^5}\right]\Bigg|_{u=8}^{u=5t-2}$$

$$= -\frac{1}{25}\left[\frac{1}{(5t-2)^5} - \frac{1}{8^5}\right] = \frac{1}{25}\left[\frac{1}{8^5} - \frac{1}{(5t-2)^5}\right]$$

$$\int_2^{\infty} \frac{dx}{(5x-2)^6}\,dx = \lim_{t\to\infty} \frac{1}{25}\left[\frac{1}{8^5} - \frac{1}{(5t-2)^5}\right] = \boxed{\frac{1}{25}\,\frac{1}{8^5}}$$ ✓

#2 Determine whether $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(5n-2)^6}$ converges or diverges

Sol: The function $f(x) = \dfrac{1}{(5x-2)^6}$ is continuous, positive, and decreasing on $[2,\infty)$.

Previous problem tells us $\displaystyle\int_2^{\infty} f(x)\,dx$ converges.

So $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(5n-2)^6}$ is convergent by the Integral Test.

#3 (i) Evaluate $\int_1^{\infty} x^2 e^{-x^3}\,dx$

Sol: $\int_1^t x^2 e^{-x^3}\,dx = \int_{u=-1}^{u=-t^3} e^u \left(-\frac{1}{3}\right) du = -\frac{1}{3} e^u \Big|_{u=-1}^{u=-t^3}$

$u = -x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}\,du = x^2\,dx$

$= -\frac{1}{3}\left(e^{-t^3} - e^{-1}\right)$

$= \frac{1}{3}\left[\frac{1}{e} - \frac{1}{e^{t^3}}\right]$

$$\int_1^{\infty} x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \int_1^t x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \frac{1}{3}\left[\frac{1}{e} - \frac{1}{e^{t^3}}\right] = \boxed{\frac{1}{3}\frac{1}{e}}$$

So $\int_1^{\infty} x^2 e^{-x^2}\,dx$ converges

(ii) Determine whether $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges or diverges

Sol: $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ converges by the Integral Test

#4 (i) Evaluate $\displaystyle\int_2^{\infty} \frac{1}{x[\ln(4x)]^2}\, dx$

Sol: $$\int_2^{t} \frac{1}{x}\frac{1}{[\ln(4x)]^2}\, dx = \int_{u=\ln(8)}^{u=\ln(4t)} \frac{1}{u^2}\, du = -\frac{1}{u}\Bigg|_{\ln 8}^{\ln t} = -\frac{1}{\ln 4t} + \frac{1}{\ln 8}$$

$u = \ln(4x)$

$du = \frac{1}{4x}\cdot 4\, dx$

$$\int_2^{\infty} \frac{1}{x}\frac{1}{[\ln(4x)]^2}\, dx = \lim_{t\to\infty} \int_{\ln 8}^{\ln t} \frac{1}{x}\frac{1}{[\ln(4x)]^2}\, dx$$

$$= \lim_{t\to\infty} -\frac{1}{\ln 4t} + \frac{1}{\ln 8}$$

$$= \frac{1}{\ln 8}$$

(ii) Determine whether $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n}\frac{1}{(\ln(4n))^2}$ converges or diverges

Sol: $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n}\frac{1}{(\ln(4n))^2}$ $\boxed{\text{converges}}$ by the Integral Test