

$0! = 1$ and if $n > 0$ then $n! = 1 \times 2 \times 3 \times \dots \times n$

The n -th Term Test for Divergence: $\sum a_n$ diverges if $\lim_{n \rightarrow \infty} a_n$ fails to exist or is different from zero.

The Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with positive terms and suppose

$$L = \lim_{n \rightarrow \infty} \frac{a_n}{b_n}.$$

- If L is finite and $L > 0$, then the series both converge or both diverge.
- If $L = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- If $L = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Ratio Test: Let $\sum a_n$ be a series with nonzero terms and suppose $\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$.

- If $\rho < 1$, the series converges absolutely.
- If $\rho > 1$ or $\rho = \infty$, the series diverges.
- If $\rho = 1$, then the test is inconclusive, use a different test.

Alternating Series Test: An alternating series $\sum (-1)^n u_n$ or $\sum (-1)^{n+1} u_n$ converges if the following three conditions are satisfied:

- $u_n > 0$ for all $n \geq N$
- $\lim_{n \rightarrow \infty} u_n = 0$
- $u_n \geq u_{n+1}$ for all $n \geq N$ for some N