

Sec 7.5

Def: Elementary functions

An elementary function is a function which can be obtained from

- power functions (like $x^{\frac{1}{5}}$),
- exponential functions (like e^x),
- logarithmic functions (like \ln),
- trig functions (like $\sec(x)$),
- inverse trig functions (like $\operatorname{arcsec}(x)$)

by addition/subtraction/multiplication/division/composition

Example

$$f(x) = \sqrt{x^3+1} + \frac{1}{\ln x} - e^{(x^2)}$$

• If f is an elementary function, then f' is also an elementary function

but $\int f(x) dx$ may not be an elementary function.

— In fact, the majority of elementary functions do not have elementary antiderivatives

(and we cannot compute them using u -substitution or Ch 7 methods)

These integrals cannot be expressed as elementary functions:

$$\int e^{x^2} dx \quad \int \frac{e^x}{x} dx \quad \int \sin(x^2) dx \quad \int \cos(e^x) dx$$

$$\int \sqrt{x^3+1} dx \quad \int \frac{1}{\ln x} dx \quad \int \frac{\sin x}{x} dx$$

We will learn how to evaluate these in Sec 11.9-11.10

Strategy for Integration

Simplify

• e.g. $\int \frac{1+x}{\sqrt{x}} dx = \int \left(x^{-\frac{1}{2}} + x^{\frac{1}{2}} \right) dx$

• e.g. $\int \tan x (\cos x)^2 dx = \int \frac{\sin x}{\cos x} (\cos x)^2 dx$
 $= \int \sin x \cos x dx$

Try u-sub

• e.g. $\int \sin x \cos x dx$, do $u = \sin x$ or $u = \cos x$
 $du = \cos x dx$ $du = -\sin x dx$

• e.g. $\int \frac{x}{\sqrt{x^2-1}} dx$, do $u = x^2-1$
 $du = 2x dx$

Classify by type of function

• product of trig functions, do methods in Sec 7.2 e.g. $\int (\cos x)^4 dx$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

• A rational function, see Sec 7.4 e.g. $\int \frac{1}{(x+1)(x-2)} dx$

• Radicals $\sqrt{(ax+b)^2 + c^2}$ or $\sqrt{(ax+b)^2 - c^2}$ or $\sqrt{c^2 - (ax+b)^2}$
 use trig substitution, see Sec 7.3 e.g. $\int \frac{1}{(x^2-2)^{\frac{3}{2}}} dx$

• Radicals $(ax+b)^{\frac{1}{n}}$ like $\sqrt{ax+b}$ or $\sqrt[3]{ax+b}$ e.g. $\int \frac{1}{x+\sqrt{x}} dx$
 use Rationalizing substitution $u = (ax+b)^{\frac{1}{n}}$, see end of Sec 7.4

• product of x^{\square} , a trig function, an exponential, or log function
 e.g. $\int e^x \sin x dx$, $\int x^2 e^x dx$

• an inverse trig function or a log function
 $\int \ln x dx$, $\int \arcsin x dx$

try Integration by Parts, see Sec 7.1

(the functions in Webwork 7.1 are of this type)