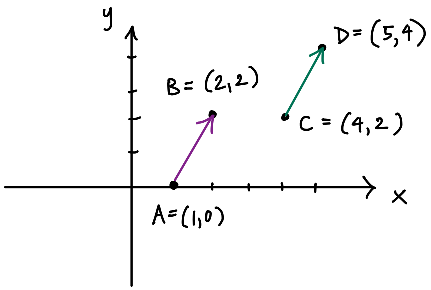
Def (vector)

A vector is a quantity with length (magnitude) and direction.



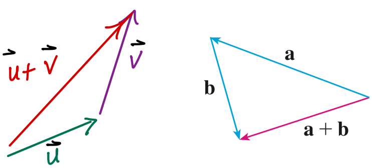
- The vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent.
- The zero vector, denoted $\mathbf{0}$ or $\vec{0}$. has length zero and no direction.

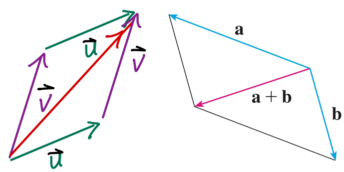
bold text

Def (vector addition)

To get the vector $\vec{u}+\vec{v}$,

put the initial point of v at the terminal point of u.





Then $\vec{u}+\vec{v}$ is the vector

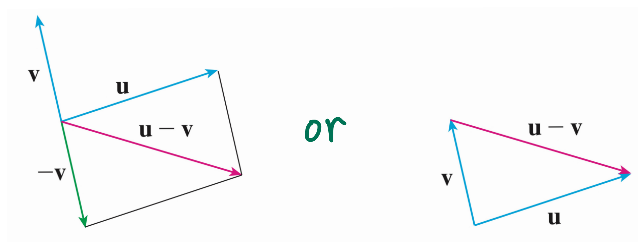
from the initial point of $\vec{u}$

to the terminal point of $\vec{v}$

Fact:

$$\vec{u}+\vec{v} = \vec{v}+\vec{u}$$

Vector subtraction:



Def (scalar multiplication)

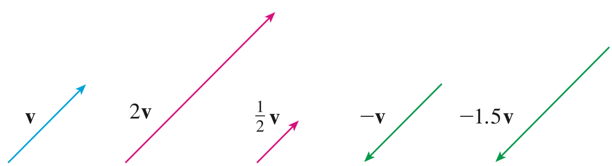
Def A **scalar** is a number

Def If $\vec{v}$ is a vector and c is a number,

the **scalar multiple** $c\vec{v}$ is a vector with ...

- length $|c|$ times length of $\vec{v}$
- (if $c > 0$) same direction as $\vec{v}$
  (if $c < 0$) opposite direction to $\vec{v}$
- If $c = 0$ or $\vec{v} = \vec{0}$, then $c\vec{v} = \vec{0}$.

Scalar multiples of $\vec{v}$



The numbers $1, 2, \frac{1}{2}, -1, -1.5$ "scale" the vector $\vec{v}$.

That's why they are called scalars.

Note: Two vectors are parallel (having the same or opposite direction) if they are scalar multiples of one another.

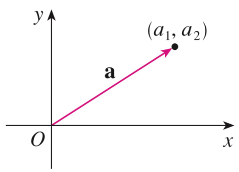
Components

Place the initial point of a vector $\vec{a}$ at the origin.
Then the terminal point of $\vec{a}$ has coordinates
(a_1, a_2) or (a_1, a_2, a_3).

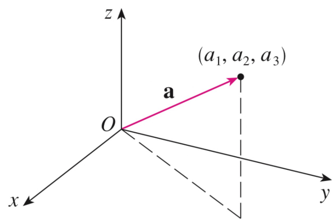
These coordinates are called the **components** of **a** and we write

$$\mathbf{a} = \langle a_1, a_2 \rangle \qquad \text{or} \qquad \mathbf{a} = \langle a_1, a_2, a_3 \rangle$$

We use the notation $\langle a_1, a_2 \rangle$ for the ordered pair that refers to a vector so as not to confuse it with the ordered pair (a_1, a_2) that refers to a point in the plane.



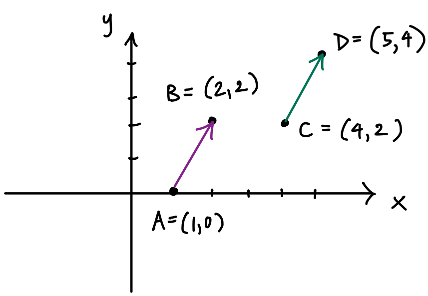
$\mathbf{a} = \langle a_1, a_2 \rangle$



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

Webwork Problems 1, 2, 3, 4

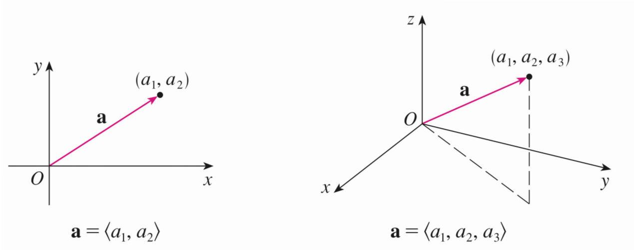
Find a vector with representation given by $\overrightarrow{CD}$



Answer $\langle 5-4,\ 4-2 \rangle$

$= \langle 1, 2 \rangle$

Length of a vector



The length of the two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

The length of the three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Use the distance formula

Webwork Problem 11

Find the length of the vector $\mathbf{v} = \langle 10, 6, 3 \rangle$.

$\|\mathbf{v}\|$ = ________

$$\sqrt{10 \cdot 10 + 6 \cdot 6 + 3 \cdot 3} = \sqrt{145}.$$

Webwork Problems 16, 17

Let $\vec{a} = \langle -10, 9, 10 \rangle$.

- Find a unit vector in the same direction as $\vec{a}$

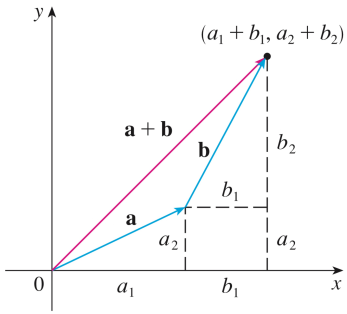
- Find a vector that has the same direction as $\vec{a}$ but has length 3.

Answer

The given vector has length $|\langle -10,9,10\rangle| = \sqrt{(-10)^2+9^2+10^2} = \sqrt{281}$, so a unit vector in the direction of $\langle -10,9,10\rangle$ is $\mathbf{u} = \frac{1}{\sqrt{281}}\langle -10,9,10\rangle$.
A vector in the same direction but with length 3 is

$$3\mathbf{u} = \frac{3}{\sqrt{281}}\langle -10,9,10\rangle = \left\langle \frac{-30}{\sqrt{281}}, \frac{27}{\sqrt{281}}, \frac{30}{\sqrt{281}} \right\rangle.$$

Adding vectors using algebra



If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

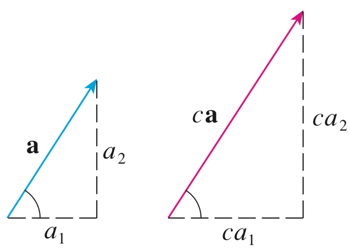
$$\mathbf{a} + \mathbf{b} = \langle \underbrace{a_1 + b_1}, \underbrace{a_2 + b_2} \rangle$$

Add each component of $\vec{a}$ and $\vec{b}$

If $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$,

$$\begin{aligned} \mathbf{a} + \mathbf{b} &= \langle 4, 0, 3 \rangle + \langle -2, 1, 5 \rangle \\ &= \langle 4 + (-2), 0 + 1, 3 + 5 \rangle = \langle 2, 1, 8 \rangle \end{aligned}$$

Scaling vector using algebra



$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Multiply each Component of $\vec{a}$ by c

If $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$,

$$\begin{aligned} 2\mathbf{a} + 5\mathbf{b} &= 2\langle 4, 0, 3 \rangle + 5\langle -2, 1, 5 \rangle \\ &= \langle 8, 0, 6 \rangle + \langle -10, 5, 25 \rangle = \langle -2, 5, 31 \rangle \end{aligned}$$

Webwork Problem 13

Let $\overline{u} = \langle 1, 1\rangle$, $\overline{v} = \langle 5, -1\rangle$, and $\overline{w} = \langle -4, 0\rangle$. Find the vector $\overline{x}$ that satisfies

$$10\overline{u} - \overline{v} + \overline{x} = 8\overline{x} + \overline{w}.$$

Answer:

$$10\vec{u} - \vec{v} - \vec{w} = 7\vec{x}$$

$$\frac{1}{7}\left(10\vec{u} - \vec{v} - \vec{w}\right) = \vec{x}$$

$$\vec{x} = \frac{1}{7}\left(\langle 10.1, 10.1\rangle - \langle 5, -1\rangle - \langle -4, 0\rangle\right)$$

$$= \frac{1}{7}\left(\langle 10 - 5 + 4, 10 + 1\rangle\right)$$

$$= \frac{1}{7}\langle 9, 11\rangle$$

$$= \boxed{\left\langle \frac{9}{7}, \frac{11}{7} \right\rangle}$$