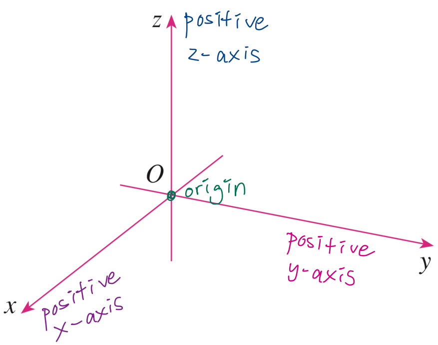
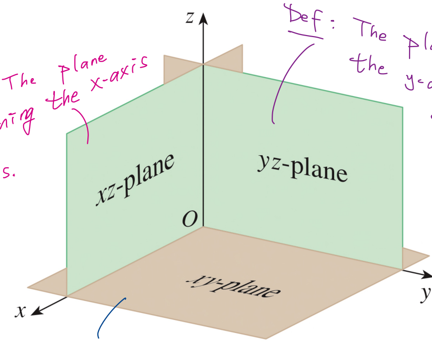
Part I: 3D space

To represent a point in 3D, draw 3 coordinate axes.



Def: The plane containing the x-axis and z-axis.

Def: The plane containing the y-axis and the z-axis.

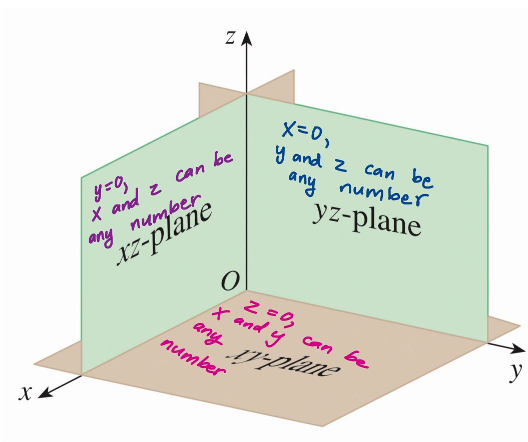


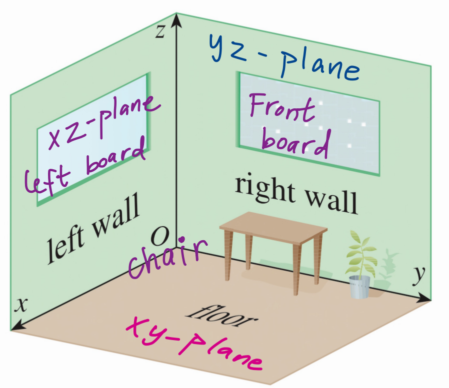
Def: The xy-plane is the (horizontal) plane containing the x-axis and the y-axis

---

The three planes divide space into eight octants.

The first octant is determined by the positive axes.





Part I: 3D space

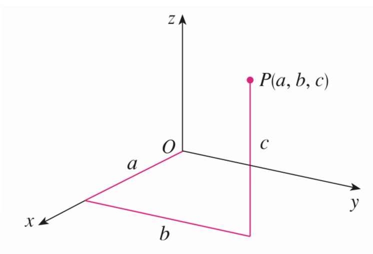
To find the point (a, b, c) in 3D space,

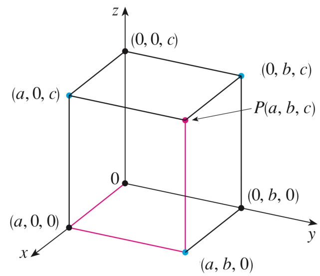
start from the origin O

Move a units along the x-axis

Move b units parallel to the y-axis

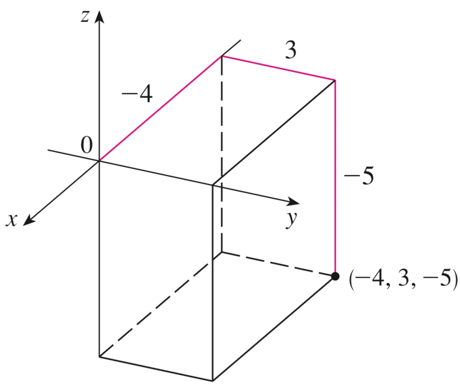
Move c units parallel to the z-axis



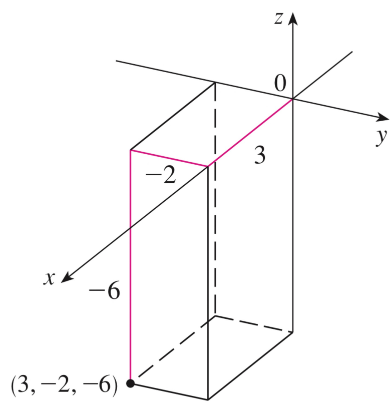


Examples:

point (-4, 3, -5)



point (3, -2, -6)



Part II: Planes

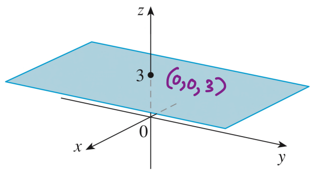
EXAMPLE 1 What surfaces in $\mathbb{R}^3$ are represented by the following equations?

(a) $z = 3$ (b) $y = 5$

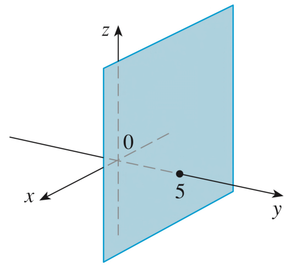
The set of points $(x, y, 3)$ where x and y can be any number

The set of all points (x,y,z) in 3D space where $y=5$ and x, z can be any number.

Parallel to the xz-plane (the "left wall")



(a) $z = 3$, a plane in $\mathbb{R}^3$



(b) $y = 5$, a plane in $\mathbb{R}^3$

Parallel to the xy-plane (the horizontal "floor")

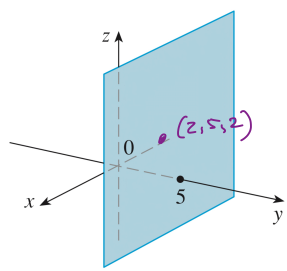
Webwork Problem 2:

An equation for the plane parallel to the xz-plane and passing through the point $(2,5,2)$ is

y

$\boxed{y=5}$ x, z can be any number

If we replace "xz-plane" with "yz-plane", the answer would be $\boxed{x=2}$



(b) $y = 5$, a plane in $\mathbb{R}^3$

Part II: Planes

EXAMPLE 3 Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $y = x$.

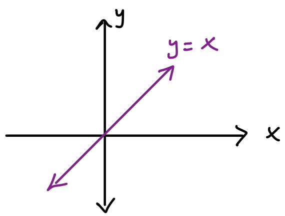
Go to geogebra.org/3d and type $y = x$

The surface is the set of points $(x, y, z) = (x, x, z)$ where x and z are any numbers.

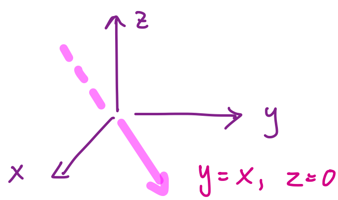
We get a plane containing:

* the z-axis (since $(0,0,z)$ is in the surface for any z)
* the line $y = x$, $z = 0$

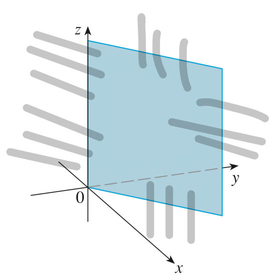
Step 1: Sketch



Step 2: Draw on the floor:

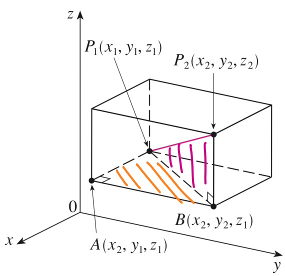


Step 3: Allows any value for z to get an "infinite wall"



Plane $y = x$

Part III : Distance

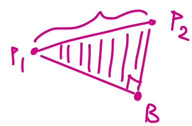


Distance Formula in Three Dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

why ?

- Apply Pythagorean Theorem to find $|P_1 B|$
- Apply it again for

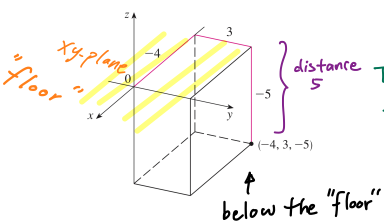


EXAMPLE 4 The distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$ is

$$|PQ| = \sqrt{(1-2)^2 + (-3+1)^2 + (5-7)^2} = \sqrt{1+4+4} = 3$$

Webwork Problem 1 :

Find the distance from $(-4, 3, -5)$ to the xy-plane.

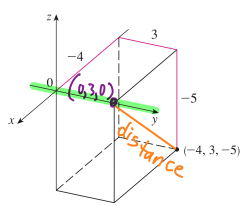


Answer: 5

The xy-plane is the horizontal "floor".

The distance is the difference between $z=0$ and $z=-5$

Find the distance from $(-4, 3, -5)$ to the y-axis.



Answer:

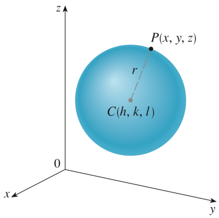
The point on the y-axis closest to $(-4, 3, -5)$ is $(0, 3, 0)$.

Distance : $\sqrt{(-4-0)^2 + (3-3)^2 + (-5-0)^2}$

$= \sqrt{16+25}$

$= \sqrt{41}$

Part IV: Spheres



The set of points of distance r from (h, k, ℓ) can be described by equation

$$r = \sqrt{(x-h)^2 + (y-k)^2 + (z-\ell)^2}, \text{ so} \ldots$$

Equation of a Sphere An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

In particular, if the center is the origin O, then an equation of the sphere is

$$x^2 + y^2 + z^2 = r^2$$

Webwork Problem 6

Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$$

Answer:

$$x^2 + 4x \quad + \quad y^2 - 6y \quad + \quad z^2 + 2z \quad = -6$$

"Complete squares" $x^2 + 4x + 2^2 + y^2 - 6y + 3^2 + z^2 + 2z + 1^2 = -6 + 4 + 9 + 1$

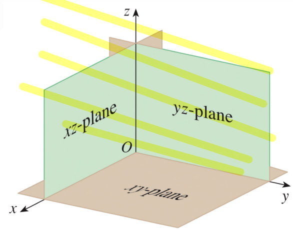
$$(x+2)^2 + (y-3)^2 + (z+1)^2 = 8$$

Comparing this equation with the standard form, we see that it is the equation of a sphere with center $\boxed{(-2, 3, -1)}$ and radius $\boxed{\sqrt{8} = 2\sqrt{2}}$.

Webwork Problem 4 (a)

Consider the sphere $(x-3)^2+(y-5)^2+(z-4)^2=25$

(a) Does the sphere intersect each of the following planes at zero points, at one point, at two points, in a line, or in a circle?



i. The sphere intersects the yz-plane ?

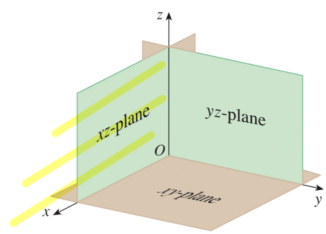
- yz-plane is the collection of points $(0, y, z)$
- Set $x=0$: $(-3)^2 + (y-5)^2 + (z-4)^2 = 25$

$$\left.(y-5)^2 + (z-4)^2 = 16\right\}$$ a circle

The sphere intersects the yz-plane

✓ $\boxed{\text{in a circle}}$

ii. The sphere intersects the xz-plane ?



Answer: The xz-plane is the set of points $(x, 0, z)$ for any x, z.

Set : $(x-3)^2 + (-5)^2 + (z-4)^2 = 25$

$$(x-3)^2 + (z-4)^2 = 0$$

This equation is satisfied for $x=3$, $z=4$

So the sphere intersects the xz-plane at exactly $\boxed{\text{one point.}}$

This point is $(x=3, y=0, z=4)$

Webwork Problem 4 (b)

Consider the sphere $(x-3)^2+(y-5)^2+(z-4)^2=25$

(b) Does the sphere intersect each of the following coordinate axes at zero points, at one point, at two points, or in a line?

The sphere intersects the z-axis $\boxed{?}$

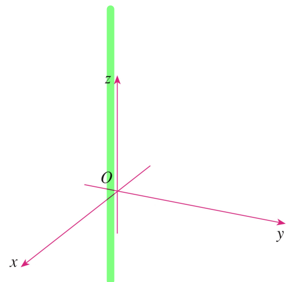


FIGURE 1
Coordinate axes

The z-axis is the set of points $(0,0,z)$, z any number

Set $x=y=0$: $(-3)^2+(-5)^2+(z-4)^2=25$

$$(z-4)^2=-9$$

No z satisfies this equation

The sphere does <u>not</u> intersect the z-axis