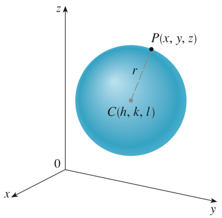
12.1



Equation of a Sphere An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

In particular, if the center is the origin O, then an equation of the sphere is

$$x^2 + y^2 + z^2 = r^2$$

Webwork # 6

Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$$

Answer: $x^2 + 4x \quad + \quad y^2 - 6y \quad + \quad z^2 + 2z \quad = -6$

"Complete squares" $x^2 + 4x + 2^2 + y^2 - 6y + 3^2 + z^2 + 2z + 1^2 = -6 + 4 + 9 + 1$

$$(x + 2)^2 + (y - 3)^2 + (z + 1)^2 = 8$$

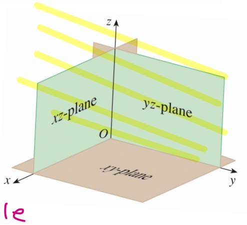
Comparing this equation with the standard form, we see that it is the equation of a sphere with center $\boxed{(-2, 3, -1)}$ and radius $\boxed{\sqrt{8} = 2\sqrt{2}}$.

12.1 Webwork # 4

Consider the sphere $(x-3)^2+(y-5)^2+(z-4)^2=25$

(a) Does the sphere intersect each of the following planes at zero points, at one point, at two points, in a line, or in a circle?

i. The sphere intersects the yz-plane ?

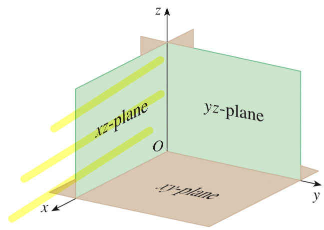


- yz-plane is the collection of points $(0, y, z)$
- Set $x=0$: $(-3)^2 + (y-5)^2 + (z-4)^2 = 25$

$$\left.(y-5)^2 + (z-4)^2 = 16 \right\} \text{ a circle}$$

The sphere intersects the yz-plane $\boxed{\text{in a circle}}$

ii. The sphere intersects the xz-plane ?



Answer: The xz-plane is the set of points $(x, 0, z)$ for any x, z.

Set : $(x-3)^2 + (-5)^2 + (z-4)^2 = 25$

$$(x-3)^2 + (z-4)^2 = 0$$

This equation is satisfied for $x=3$, $z=4$

So the sphere intersects the xz-plane at exactly $\boxed{\text{one point.}}$

(b) Does the sphere intersect each of the following coordinate axes at zero points, at one point, at two points, or in a line?

The sphere intersects the z-axis ?

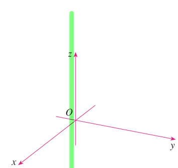


FIGURE 1
Coordinate axes

The z-axis (z) is the set of points $(0,0,z)$, z any number

Set $x=y=0$: $(-3)^2 + (-5)^2 + (z-4)^2 = 25$

$$(z-4)^2 = -9$$

No z satisfies this equation

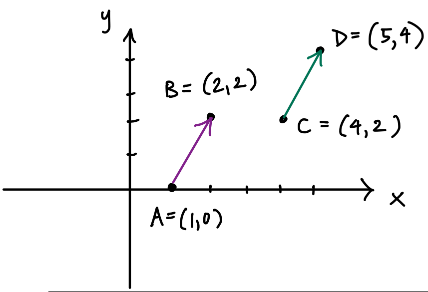
The sphere does <u>not</u> intersect the z-axis

① If $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$,

$$2\mathbf{a} + 5\mathbf{b} = 2\langle 4, 0, 3\rangle + 5\langle -2, 1, 5\rangle$$
$$= \langle 8, 0, 6\rangle + \langle -10, 5, 25\rangle = \langle -2, 5, 31\rangle$$

---

② Find a vector with representation given by $\overrightarrow{CD}$



Answer $\langle 5-4, 4-2 \rangle$

$= \langle 1, 2 \rangle$

Webwork # 13

Let $\overline{u} = \langle 1, 1\rangle$, $\overline{v} = \langle 5, -1\rangle$, and $\overline{w} = \langle -4, 0\rangle$. Find the vector $\overline{x}$ that satisfies

$$10\overline{u} - \overline{v} + \overline{x} = 8\overline{x} + \overline{w}.$$

$$10\vec{u} - \vec{v} - \vec{w} = 7\vec{x}$$

$$\frac{1}{7}\left(10\vec{u} - \vec{v} - \vec{w}\right) = \vec{x}$$

$$\vec{x} = \frac{1}{7}\left(\langle 10.1, 10.1\rangle - \langle 5, -1\rangle - \langle -4, 0\rangle\right)$$

$$= \frac{1}{7}\left(\langle 10-5+4, 10+1\rangle\right)$$

$$= \frac{1}{7}\langle 9, 11\rangle = \boxed{\left\langle \frac{9}{7}, \frac{11}{7}\right\rangle}$$

12.3

① $\langle 2, 4\rangle \cdot \langle 3, -1\rangle = \quad 2(3) + 4(-1) = 2$

$\langle -1, 7, 4\rangle \cdot \left\langle 6, 2, -\tfrac{1}{2}\right\rangle = \quad (-1)(6) + 7(2) + 4\left(-\tfrac{1}{2}\right) = 6$

---

② **3 Theorem** If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

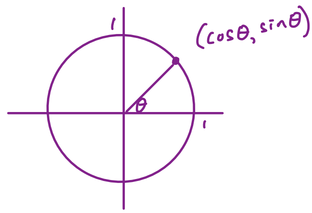
---

③

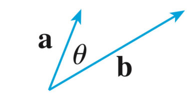
$\cos(\theta)$ is ... positive when $0 \le \theta < \frac{\pi}{2}$

0 when $\theta = \frac{\pi}{2}$

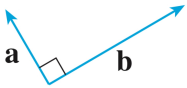
negative when $\frac{\pi}{2} < \theta \le \pi$



So we can determine acute / right / obtuse angle by computing $\vec{a} \cdot \vec{b}$

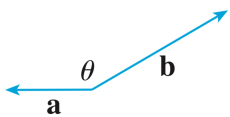


$\mathbf{a} \cdot \mathbf{b} > 0$
θ acute



$\mathbf{a} \cdot \mathbf{b} = 0$
$\theta = \pi/2$

Two vectors $\vec{a}, \vec{b}$ are orthogonal or perpendicular if $\vec{a} \cdot \vec{b} = 0$



$\mathbf{a} \cdot \mathbf{b} < 0$
θ obtuse

---

④ Is the angle between $\langle 2,4\rangle$ and $\langle 3,-1\rangle$ acute, obtuse, or $0, \pi$?

Since they are not scalar multiples of each other, the angle between them is between 0 and π (excluding $0, \pi$). Since $\langle 2,4\rangle \cdot \langle 3,-1\rangle = 2$ is positive, we must have $0 < \theta < \frac{\pi}{2}$, an acute angle

12.4

1. Compute the following determinant.

$$\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 1(7) - 3(2) = 1$$

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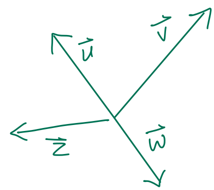
2. The following is a sketch of four vectors $\vec{u}, \vec{v}, \vec{w}, \vec{z}$.



Does $\vec{u} \times \vec{v}$ point in or out of the screen? In

Does $\vec{u} \times \vec{z}$ point in or out of the screen? out

Does $\vec{u} \times \vec{w}$ point in or out of the screen?

Neither. $\vec{u} \times \vec{w}$ is the zero vector, since $\vec{u}$ and $\vec{w}$ are parallel

---

3. Select the true statement.

- ☐ ~~$\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$ for all vectors $\vec{a}, \vec{b}$~~ false
- ☐ $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ for all vectors $\vec{a}, \vec{b}$ True

---

4. If $\vec{v} \times \vec{w} = \vec{0}$, that means ...

- ☐ $\vec{v}$ and $\vec{w}$ are parallel
- ☐ $\vec{v}$ and $\vec{w}$ are perpendicular