10.1 (Webwork #14)

Eliminate the parameter to express

$x = e^{3t}$

$y = \dfrac{1}{e^{6t}}$ in the form $y = f(x)$

Answer: $y = \dfrac{1}{(e^{3t})^2} = \dfrac{1}{x^2}$ Ans: $\boxed{y = \dfrac{1}{x^2}}$

Similar Webwork practice: # 15, 16, 17, 18, 20

Webwork Sketching parametric curve: # 10, 12

10.2 (Webwork # 10)

Useful fact: $\dfrac{d^2y}{dx^2} = \dfrac{\left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right]}{\left[\frac{dx}{dt}\right]}$

Suppose $h(t)$ and $k(t)$ are differentiable functions of t. Consider the parametric curve

$\begin{cases} x = h(t) \\ y = k(t) \end{cases}$, where y is also a differentiable function of x.

Suppose you have computed $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = \dfrac{-6}{(t^2-4)^2}$ and $\dfrac{dx}{dt} = 3t^2 - 12$

List the t-interval where the curve is concave upward.

$\boxed{?} < t < \boxed{?}$

Answer:

The curve is concave up when $\dfrac{d^2y}{dx^2}$ is defined and is a positive number.

$$\frac{d^2y}{dx^2} = \frac{\left[\frac{d}{dt}\left(\frac{dy}{dx}\right)\right]}{\left[\frac{dx}{dt}\right]} = \frac{\left[\frac{-6}{(t^2-4)^2}\right]}{\left[3t^2-12\right]} = -\frac{6}{3}\,\frac{1}{(t^2-4)(t^2-4)^2} = \frac{-2}{(t^2-4)^3}$$

$\dfrac{d^2y}{dx^2}$ is positive when $(t^2-4)^3 < 0$

$\iff t^2 - 4 < 0$

$\iff t^2 < 4$

Answer $\boxed{-2 < t < 2}$

Similar Webwork practice: # 8, 9, 10, 12

10.3

1. Determine the polar coordinates of the two points at which the polar curves $r = 7\sin(\theta)$ and $r = 7\cos(\theta)$ intersect. Restrict your answers to $r \geq 0$ and $0 \leq \theta < 2\pi$.

To input answers, list the two points in order of increasing values of r. If both points have the same value of r,, list them in order of increasing values of θ. If one of the intersection points is the pole, type "pole" in lower-case letters in both blanks for the first point.

Intersection point 1: $(r, \theta) =$ (pole, pole)
Intersection point 2: $(r, \theta) = \left(\frac{7\sqrt{2}}{2}, \frac{\pi}{4}\right)$

Set $7\sin\theta = 7\cos\theta \Rightarrow \boxed{\theta = \frac{\pi}{4}}$ and $\boxed{r = 7\cos\frac{\pi}{4} = 7\frac{\sqrt{2}}{2}}$

Also when $r = 0$, the pole

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2. Consider the curves $r = 4$ and $r\cos\theta = 4$.

At how many points do they intersect? Ans: 1

At what point/s do they intersect? Ans: $(r = 4, \theta = 0)$

$r = 4$ is the circle with radius 4 centered at the origin.

$r\cos\theta = 4$ is equivalent to the Cartesian equation $x = 4$,

which describes the vertical line through the point $(x = 4, y = 0)$

So the only intersection point is at $(x = 4, y = 0)$

(equivalently, at $(r = 4, \theta = 0)$ in polar coordinates)

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3. Consider the curves $\theta = \frac{\pi}{6}$ and $(x-4)^2 + (y+2)^2 = 1$.

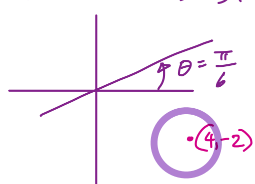
At how many points do they intersect? Ans: 0

At what point/s do they intersect? N/A

$\theta = \frac{\pi}{6}$ is the line through the origin that makes an angle $\frac{\pi}{6}$ with the positive x-axis.

$(x-4)^2 + (y+2)^2 = 1$ is the circle with radius 1 centered at $(4, -2)$.

This circle is in the 4th quadrant, so there are no intersection points.



10.4

See Quiz 6 Study Guide