① Memorize the statement of the Limit Comparison Test (for when $\lim_{n\to\infty} \frac{a_n}{b_n}$ is a positive number)

② For $a_n = \frac{7}{5^n - 2}$, find a sequence b_n so that

$\lim_{n\to\infty} \frac{a_n}{b_n}$ is a positive (finite) number

Then apply the Limit Comparison Test to $\sum_{n=1}^{\infty} a_n$.

Sec 11.4 Review pg 2

(3) Use the Limit Comparison Test to test $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^8 - 6}}$

for convergence/divergence.

Sec 11.4 Review pg 3

(4) Using the **Limit Comparison Test**, determine if the series $\sum_{n=1}^{\infty} \frac{n^4 - 2n^2 + 3}{2n^6 - n + 5}$ converges.

(5) Is $\sum_{n=1} \frac{\sqrt{n+1}}{n}$ convergent or divergent ?

Sec 11.4 Review pg 4

6 Test the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ for convergence or divergence.

7 Determine whether the series $\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{\sqrt{5 + n^5}}$ converges or diverges.

Memorize the statement of the Alternating Series Test pg 772

Determine if the following series converge or diverge, with justification.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ (Copy solution from Example 1, pg 774)

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$ (Copy solution from Example 3, pg 774)

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{3n}{4n-1}$ (Copy solution from Example 2, pg 774)

Sec 11.5 Review pg 2

(d) Determine whether the series $\sum_{n=1}^{\infty} \frac{-n^2 \cos(n\pi)}{n^3+1}$ converges/diverges.

(e) Determine whether the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{-n}$ converges/diverges.

(f) Is $\sum_{n=2}^{\infty} \frac{(\sqrt{n}-1)\cos(n\pi)}{\sqrt{n^2-3}}$ an alternating series ?

1.) Memorize the statement of the Ratio Test pg 779

Determine whether each of the following is conditionally convergent, absolutely convergent, or divergent.

2.)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

3.)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

Sec 11.6 Review pg 3

Determine whether each of the following is conditionally convergent, absolutely convergent, or divergent.

4.) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

5.) $\sum_{k=1}^{\infty} \frac{10^k}{k!}$

WEBWORK PROBLEM 1, 2, 7, 8, 11, 12

Determine whether each of the following is conditionally convergent, absolutely convergent, or divergent.

6.) $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$

7.) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

Sec 11.7 Review pg 1

For each of the following series, determine whether the ratio test will work for testing convergence / divergence.

① $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{(n^2+4)}$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

② $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^5}$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

③ $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(\pi n)+6\sqrt{n}}{n^2}$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

④ $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+1)!\,2}$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

⑤ $\displaystyle\sum \frac{(-2)^n}{3^n} \frac{\ln n}{(n^2+4)}$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

⑥ $\displaystyle\sum \frac{\sqrt{n}+1}{n!} (-1)^n$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

⑦ $\displaystyle\sum_{n=2}^{\infty} \frac{6}{n\sqrt{n^7-2}}$

- ○ The ratio test can be used
- ○ The ratio test will be inconclusive

Key points from textbook pg 784-785

8

- Q: When will Ratio Test be inconclusive ?

9

- Q: When is Ratio Test likely to work ?

10

- Q: When can Limit Comparison Test work ?

WEBWORK PROBLEM 1

WEBWORK PROBLEM 4

Sec 11.8 Review pg 1

Q: What theorem should you use to find the radius of convergence of a power series?

Q: What tests should you use to check whether $x = a + R$ and $x = a - R$ (where a is the center of the power series and R is the radius) are in the interval of convergence?

Q: If $\lim_{n\to\infty} \left| \frac{C_{n+1} (x-10)^{n+1}}{C_n (x-10)^n} \right| = \left| \frac{x-10}{99} \right|$, what is the radius of convergence for $\sum_{n=1}^{\infty} C_n (x-10)^n = c_1 (x-10) + c_2 (x-10)^2 + c_3 (x-10)^3 + \dots$? What about interval of convergence?

WEBWORK PROBLEM 2, 6, 9, 10

Sec 11.8 Review Pg 2

Find the radius of convergence and interval of convergence for each series.

$\boxed{1.}$ $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{5^n} = 1 + \frac{x+2}{5} + \frac{(x+2)^2}{25} + \ldots$

$\boxed{2.}$ $\displaystyle\sum_{n=0}^{\infty} 9^n (x-2)^{2n}$

$\boxed{3.}$ Find the radius of convergence and interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{5^n (x-4)^n}{\sqrt{n}}$

Sec 11.8 Review

Find the radius of convergence and interval of convergence for each series.

$\boxed{4.}$ $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\boxed{5.}$ $\displaystyle\sum_{n=0}^{\infty} n!(x+8)^n$

$\boxed{6.}$

Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -3$ and diverges when $x = 4$.
Determine whether the following series converge or diverge.

1. $\sum_{n=1}^{\infty} c_n$

2. $\sum_{n=1}^{\infty} c_n 9^n$

3. $\sum_{n=1}^{\infty} c_n (-2)^n$

4. $\sum_{n=1}^{\infty} (-1)^n c_n 12^n$

1 Find a power series representation for $f(x) = \dfrac{5}{1+4x^2}$ and find its interval of convergence.

2 Find a power series representation for $f(x) = \dfrac{2x^4}{2-3x}$ and find its interval of convergence.

3 Find a power series representation (centered at 0) for $f(x) = \dfrac{1}{(5+x)^2}$.

(4) Find the antiderivative of the power series

$$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + \cdots \quad \text{for } |x| < 1$$

(5) Find $\int \ln(1+t^4)\,dt$ as a power series, and find its radius of convergence.

(6) Use the fact $\arctan(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ (with radius of convergence 1) to find a power series representation of $\int \frac{\arctan(2x)}{x}\,dx$. Find its radius of convergence.

← impossible to solve using Chapter 7 methods

The boxed equations are from Table 1, pg 808 (will be printed for you).
Fill in the blanks.

- $\boxed{\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n} \text{ for } -1 \le x < 1.}$

1 ➢ $\sum_{n=1}^{\infty} \frac{1}{n\,2^n} =$ ________ .

2 • $\ln(1+x) =$ ____________________ for ____________ .

- $\boxed{\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \text{ for } -1 \le x \le 1.}$

3 ➢ $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} =$ ________ .

- $\boxed{e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \text{ for } -\infty < x < \infty.}$

5 Find the sum of $1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \cdots$

4 ➢ $\sum_{n=0}^{\infty} \frac{1}{n!} =$ ________ .

- $\boxed{\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \text{ for } -\infty < x < \infty.}$

6 ➢ $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!} =$ ________ .

- $\boxed{\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \text{ for all } x}$.

7 $\sum_{n=0}^{\infty} \frac{(-1)^n}{6^{2n}} \frac{\pi^{2n}}{(2n)!} =$ ________

Sec 11.10 Review pg 2

Theorem (given for you)
IF f has a power series representation at $x = a$, that is, if

$$f(x) = \underbrace{\sum_{n=0}^{\infty} c_n (x-a)^n}_{\text{called "the Taylor series of } f(x) \text{ centered at } a\text{"}} \quad \text{for} \quad |x-a| < R,$$

THEN its coefficients are given by

$$c_n = \frac{f^{(n)}(a)}{n!}.$$

8. If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ and $f(0) = 14, \quad f'(0) = -15, \quad f''(0) = -1, \quad f'''(0) = -1$,

find the first four terms of $\sum_{n=0}^{\infty} c_n x^n$.

---

9. (From Textbook Example 1) If $f^{(n)}(a) = 3^n$ for all $n \geqslant 0$,

what is the Taylor series of $f(x)$ centered at $x = a$?

10 • One of the Webwork problems is to find that

$$\boxed{\ln(x) = \ln(10) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{10^n n}(x-10)^n}$$

when x is in the interval of convergence of the power series.

• Assume you have done the computation showing this equality.

Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{10^n n}(x-10)^n$.

Sec 11.10 Review pg 4

Table 1

Important Maclaurin Series and Their Radii of Convergence

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$	$R = \infty$
$\text{Arc}\tan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$	$R = 1$

11

(i) Evaluate $\int \frac{1}{2x}\,dx$ (write as a "usual" elementary function)

12

(ii) Use part (i) and Table 1 above to write $\int \frac{e^{9x}-1}{2x}\,dx$ as a power series

Sec 11.10 Review pg 5

(13) Use the table to write a power series representation of $\int \arctan(x^2)\,dx$ centered at $x=0$.

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(14) Use the table to write a power series representation of $\int \arctan(x^2)\,dx$ centered at $x=0$.

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(15) Use Table 1 to write $\int x^2 \sin(x^2)\,dx$ as a power series.

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(16) Use Table 1 to write $\int \sin(x^4)\,dx$ as a power series centered at $x=0$.

Statements to memorize

☐ Alternating Series Test — Sec 11.5

Statement:

☐ Limit Comparison Test — Sec 11.4

Statement:

☐ Ratio Test — Sec 11.6

Statement:

Sec 11.2

○ a geometric series converges when ...

○ a geometric series diverges when ...

Sec 11.4

○ a p-series series converges when...

○ a p-series series diverges when...