Math 2924, Fall 2022 — Exam 3 Fact Sheet

Power series coefficients. If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$, then $c_n = \dfrac{f^{(n)}(a)}{n!}$

Table 1: Important Maclaurin Series and their Radii of Convergence

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$

Some trig facts. $\sin 2\theta = 2\sin\theta\cos\theta$

$$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta, \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta), \quad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \cos\frac{\pi}{3} = \frac{1}{2}, \quad \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Some derivatives.

$\frac{d}{dx}\sin(x) = \cos(x)$ $\qquad \frac{d}{dx}\cos(x) = -\sin(x)$ $\qquad \frac{d}{dx}\tan(x) = (\sec(x))^2$

$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$ $\qquad \frac{d}{dx}\sec(x) = \sec(x)\tan(x)$ $\qquad \frac{d}{dx}\cot(x) = -(\csc(x))^2$

$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$