# Sec 7.1 Review Part I

Look up solutions on ...

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- Find $\int x \sin(x)\, dx$ — pg 513
- Evaluate $\int \ln(x)\, dx$

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- Compute $\int x e^x\, dx$. Use it to compute $\int x^2 e^x\, dx$ — pg 514
- Calculate $\int e^x \sin(x)\, dx$.

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- Compute $\int_0^1 \frac{x}{1+x^2}\, dx$ — pg 515
- Calculate $\int_0^1 \arctan(x)\, dx$

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- Calculate $\int_0^1 \arcsin(x)\, dx$ — Lecture notes 7.1

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- Evaluate (#2) $\int \sqrt{x}\, \ln(x)\, dx$

  (#15) $\int (\ln x)^2\, dx$

  (#26) $\int_1^2 x^2 \ln(x)\, dx$

  (#27) $\int_1^5 \frac{\ln(x)}{x^2}\, dx$

  (#31) $\int_1^5 \frac{x}{e^x}\, dx$

  (#58) $\int_1^0 \left[x e^{-x} - x^2 e^{-x}\right] dx$

  (#61) $\int_0^1 2\pi x \cos\left(\frac{\pi}{2} x\right) dx$

**7.1 Recommended Textbook Practice SOLUTIONS.pdf**

## Webwork 7.1

If $g(1) = 3$, $g(5) = 10$, and $\int_1^5 g(x)\,dx = -10$, evaluate the integral $\int_1^5 xg'(x)\,dx$.

## Webwork 7.1

Suppose that $f(1) = -5$, $f(4) = -2$, $f'(1) = -7$, $f'(4) = 1$, and $f''$ is continuous. Find the value of $\int_1^4 xf''(x)\,dx$.

# Sec 7.2 Review

These identities will be given if needed:

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin x \cos x = \tfrac{1}{2}\sin 2x$$

Look up solutions on ...

| | |
|---|---|
| Strategy for $\int (\sin x)^{\text{even number}}\,dx$ ? <br> Strategy for $\int (\cos x)^{\text{even number}}\,dx$ ? | pg 521 |
| Compute $\int (\sin x)^4\,dx$, $\int_0^{\pi} (\sin x)^2\,dx$, $\int (\cos x)^2\,dx$ | pg 520 |
| Strategy for $\int (\sin x)^{\text{odd number}} (\cos x)^m\,dx$ ? <br> Strategy for $\int (\cos x)^{\text{odd number}} (\sin x)^n\,dx$ ? | pg 521 |
| Compute $\int (\cos x)^3\,dx$, $\int (\sin x)^5 (\cos x)^2\,dx$ | pg 519 |
| Compute $\int (\cos x)^7\,dx$, $\int (\sin x)^7\,dx$ | Lecture notes 7.2 |
| Compute $\int \tan x\,dx$ | Old Section 6.4 <br> pg 432 |
| Compute $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$ | Apply u-sub $u = \sin x$ <br> $du = \cos x\,dx$ |

# Sec 7.3 Review part I

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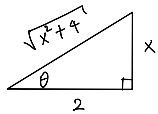
If 


then $x = \boxed{\qquad}$ $\qquad \dfrac{2}{\sqrt{x^2+4}} = \boxed{\qquad}$

$dx = \boxed{\qquad}$

$\sqrt{x^2+4} = \boxed{\qquad}$

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Set up the trig substitution for $\displaystyle\int_0^2 x^3\sqrt{x^2+4}\,dx$.

Stop there. DO NOT solve the integral.

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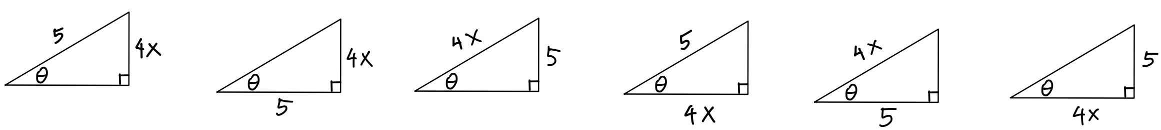
For each integral, choose all triangle/s that will work for trig substitution

I $\displaystyle\int x^2\sqrt{25-16x^2}\,dx$ $\qquad$ II $\displaystyle\int \frac{\sqrt{16x^2-25}}{x^3}$ $\qquad$ III $\displaystyle\int \frac{1}{(16x^2+25)^2}\,dx$



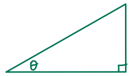

I $\qquad$ III $\qquad$ II $\qquad$ I $\qquad$ II $\qquad$ III

# Sec 7.3 Review Part II

Look up solutions on...

Compute $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,dx$ using trig substitution.

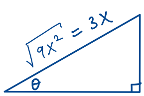
Lec Notes 7.3 A

Label all three sides of 

so that they match your trig sub.

Compute $\int \frac{1}{\left[9x^2-25\right]^{\frac{3}{2}}}\,dx$ by performing trig substitution

Lec Notes 7.3 C

using 

. (Don't use other methods)

# Sec 7.4 Review Part I

What is the partial fraction decomposition for $\frac{9x+3}{x^2-9}$ ?

(a) $\frac{A}{x-3} + \frac{B}{x+3}$

(b) $\frac{Ax+B}{x-3} + \frac{Cx+D}{x+3}$

(c) $\frac{A}{x^2} + \frac{B}{9}$

(d) $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+3} + \frac{D}{(x+3)^2}$

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What is the partial fraction decomposition for $\frac{5x^3-3x^2-8x-3}{x^4-3x^3}$

(a) $\frac{A}{x^3} + \frac{B}{x-3}$

(b) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-3}$

(c) $\frac{A}{x} + \frac{B}{x-3}$

(d) $\frac{A}{x^4} + \frac{B}{3x^3}$

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Helpful fact given: $\frac{6t}{(3t-5)(3t+5)} = \frac{1}{3t-5} + \frac{1}{3t+5}$ by partial fraction decomposition.

Question Evaluate $\int \frac{6x}{9x^2-25}\,dx$

# Sec 7.4 Review part II

True or false? It is possible to integrate every rational function (a ratio of polynomials) in terms of the functions we know

Evaluate $\int \frac{x^3 + x}{x - 1}\,dx$

Evaluate $\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x}\,dx$.

Find $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}\,dx$

Evaluate $\int \frac{\sqrt{x + 4}}{x}\,dx$

# 7.5 part I

Helpful fact given: $\dfrac{6t}{(3t-5)(3t+5)} = \dfrac{1}{3t-5} + \dfrac{1}{3t+5}$ by partial fraction decomposition. (you don't need to verify)

<u>Question</u> Describe all possible Calc 2 methods for solving $\int \dfrac{6x}{9x^2-25}\,dx$ and describe the first few steps.

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☐ The very first step is to perform partial fraction decomposition $\boxed{\quad\quad\quad\quad}$. Then use the fact that $\int \dfrac{1}{3x+b}\,dx = \boxed{\quad\quad}$

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☐ Use u-substitution with $u = \boxed{\quad\quad}$, $du = \boxed{\quad\quad}$

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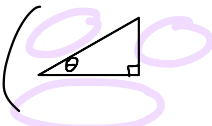
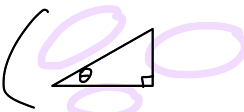
☐ Use trig substitution with (  so $\dfrac{3x}{5} = \boxed{\quad\quad}$ ) OR

(  so $\dfrac{3x}{5} = \boxed{\quad\quad}$ )

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☐ Perform Rationalizing Substitution with $u = \boxed{\quad}$, $dx\ \boxed{\quad}$ to get $\boxed{\quad\quad\quad}$ then apply partial fraction decomposition.

# 7.5 part II

Helpful fact given: $\frac{1}{(t-1)t} = \frac{1}{t-1} - \frac{1}{t}$ by partial fraction decomposition. (you don't need to verify)

<u>Question</u> Select all possible methods for solving $\int \frac{1}{1+e^x}dx$ and describe the first few steps. Leave unhelpful methods unchecked.

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☐ The very first step is to perform partial fraction decomposition $\boxed{\quad}$

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☐ Multiply $\frac{1}{1+e^x}$ by $\frac{e^{-x}}{e^{-x}}$, then do u-substitution $u = \boxed{\quad}$, $du = \boxed{\quad}$

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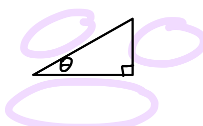
☐ Use trig substitution with  so $x = \boxed{\quad}$

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☐ Perform Rationalizing Substitution with $u = \boxed{\quad}$, $\boxed{dx =\quad}$

# 7.8 part I

1. Select all improper integrals.
Leave choices which are not improper integrals unchecked

☐ $\displaystyle\int_3^7 \frac{1}{\sqrt{x-3}}\,dx$

☐ $\displaystyle\int_3^7 \frac{e^x}{x+3}\,dx$

☐ $\displaystyle\int_6^\infty \frac{1}{x^2+3}\,dx$

☐ $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$

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2. Suppose we know that

If $t > 2$, then $\displaystyle\int_2^t f(x)\,dx = 1 - \frac{\ln(t)+1}{t}$. Evaluate $\displaystyle\int_2^\infty f(x)\,dx$.

# 7.8 part II

③ Evaluate $\int_1^\infty \frac{1}{x}\,dx$ and $\int_1^\infty \frac{1}{x^2}\,dx$.

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④ If $\int_t^5 f(x)\,dx = 2\left(\sqrt{3} - \sqrt{t-2}\right)$ for $t$ in $(2,3]$, ⓘ evaluate $\int_2^5 f(x)\,dx$.

ⓘⓘ Is $\int_2^5 f(x)\,dx$ convergent or divergent?

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⑤ If $\int_0^t f(x)\,dx = \arctan(t)$ for $t > 0$, ⓘ evaluate $\int_0^\infty f(x)\,dx$.

ⓘⓘ Is $\int_0^\infty f(x)\,dx$ convergent or divergent?

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⑥ If $\int_0^t f(x)\,dx = \ln|t-1|$ for $t$ in $[0,1)$, ⓘ evaluate $\int_0^1 f(x)\,dx$.

ⓘⓘ Is $\int_0^1 f(x)\,dx$ convergent or divergent?

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⑦ If $\int_0^t f(x)\,dx = \sin(t)$ for $t$ in $[0,\infty)$, ⓘ evaluate $\int_0^\infty f(x)\,dx$.

ⓘⓘ Is $\int_0^\infty f(x)\,dx$ convergent or divergent?

# Sec 7.8 Review part III

some helpful facts (you don't need to check):

(i) $\int_1^\infty \frac{1}{x}\,dx \overset{\text{Def}}{=} \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \ln|x|\Big|_1^t = \lim_{t\to\infty} \ln|t| - \ln(1) = \infty$

(ii) $\int_1^\infty \frac{1}{x^2}\,dx \overset{\text{Def}}{=} \lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx = \lim_{t\to\infty} 1 - \frac{1}{t} = 1$

(iii) $e^{-x}$ is positive for all $x$ in $[1,\infty)$ (in fact, for all $x$)

Question Use either $\int_1^\infty \frac{1}{x}\,dx$ or $\int_1^\infty \frac{1}{x^2}\,dx$ to show using the Comparison Test that $\int_1^\infty \frac{1+e^{-x}}{x}\,dx$ is divergent / convergent (choose one).

Answer

- Since $e^{-x}$ is always positive, we have ...

Strategies

# Sec 7.8 Review part IV

Some helpful facts (you don't need to check):

(i) $\int_1^\infty \frac{1}{x}\,dx \overset{\text{Def}}{=} \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \ln|x|\Big|_1^t = \lim_{t\to\infty} \ln|t| - \ln(1) = \infty$

(ii) $\int_1^\infty \frac{1}{x^2}\,dx \overset{\text{Def}}{=} \lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx = \lim_{t\to\infty} 1 - \frac{1}{t} = 1$

(iii) Since $0 < \arctan(x) < \frac{\pi}{2}$ for $x$ in $[1,\infty)$, we have $0 < \frac{1}{\pi}\arctan(x) < \frac{1}{2} < 1$ for $x$ in $[1,\infty)$.

Question: Use either $\int_1^\infty \frac{1}{x}\,dx$ or $\int_1^\infty \frac{1}{x^2}\,dx$ to show using the Comparison Test that $\int_1^\infty \frac{\arctan(x)}{\pi\, x^2}\,dx$ is divergence / convergent (choose one).

Answer

Strategies