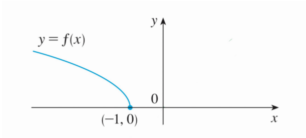
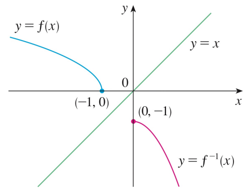
Sec 6.1 Review

① Sketch the inverse of the graph



or answer "no inverse, since f(x) is not one-to-one"

Answer:



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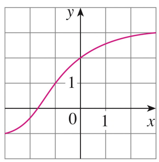
② The graph of f is given.

(a) Why is f one-to-one?
(b) What are the domain and range of f^{-1}?
(c) What is the value of $f^{-1}(2)$?
(d) Estimate the value of $f^{-1}(0)$.



(a) f is 1-1 because it passes the Horizontal Line Test.

(b) Domain of $f = [-3, 3] =$ Range of f^{-1}. Range of $f = [-1, 3] =$ Domain of f^{-1}.

(c) Since $f(0) = 2$, $f^{-1}(2) = 0$.

(d)

f seems to intersect the x-axis at x between -1.5 and -1.9, so $f^{-1}(0)$ is close to -1.7

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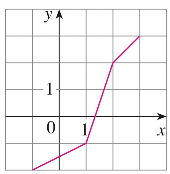
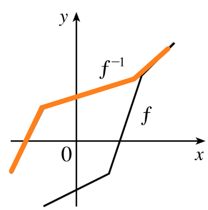
③ Sketch the inverse of the graph



or answer "no inverse"

Answer:



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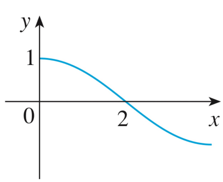
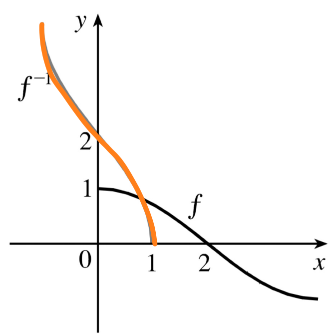
④ Sketch the inverse of the graph



or answer "no inverse"

Answer:

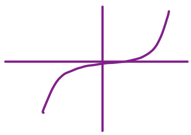


Sec 6.1 Review

Look up solutions on ...

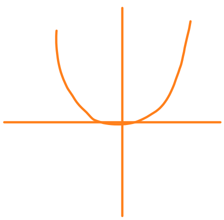
- A function f is called one-to-one if ... (if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$)

- Is $f(x) = x^3$ with domain all real numbers one-to-one? pg 401



yes

- Is $f(x) = x^2$ with domain all real numbers one-to-one?



No

- If f is one-to-one with $\underbrace{\text{domain}}_{\text{allowed inputs}}$ A and $\underbrace{\text{image/range}}_{\text{possible outputs}}$ B,

  what is the domain of the inverse function f^{-1} of f? B

  what is the image/range of the inverse function f^{-1} of f? A

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pg 402

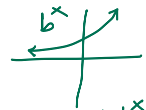
- If $f^{-1}(x) = y$, then $f(y) = \boxed{x}$
- If x is in the domain of f, then $f^{-1}(f(x)) = \boxed{x}$
- If x is in the domain of f^{-1}, then $f(f^{-1}(x)) = \boxed{x}$

Sec 6.2 Review

Look up solutions on...

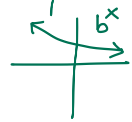
- If $b > 1$, then $\lim_{x\to-\infty} b^x = \boxed{0}$ and $\lim_{x\to\infty} b^x = \boxed{\infty}$ pg 410

  Sketch the graph $y = b^x$



- If $0 < b < 1$, then $\lim_{x\to-\infty} b^x = \boxed{\infty}$ and $\lim_{x\to\infty} b^x = \boxed{0}$

  Sketch the graph $y = b^x$



- If $b > 0$, $b^{x+y} = \boxed{b^x b^y}$ and $(b^x)^y = \boxed{b^{(xy)}}$

- If $a>0$, $b>0$, $(ab)^x = \boxed{a^x b^x}$

- Evaluate $\lim_{x\to\infty}\left[\left(\frac{1}{2}\right)^x - 1\right] = 0 - 1$

- **MEMORIZE** $\frac{d}{dx}(e^x) = \boxed{e^x}$ pg 414

- $\lim_{x\to-\infty} e^x = \boxed{0}$ (Remember) and $\lim_{x\to\infty} e^x = \boxed{\infty}$ (Remember) pg 416

- Memorize $\int e^x \, dx = \boxed{e^x + C}$ pg 417

- Evaluate $\int x^2 e^{(x^3)} \, dx$.

$u = x^3$

$du = 3x^2 \, dx$

$\frac{1}{3} du = x^2 \, dx$

$\int \frac{1}{3} e^u \, du = \frac{1}{3} e^u + C$

$= \frac{1}{3} e^{x^3} + C$

Sec 6.3 Review

Look up solutions on...

- Memorize If $\ln(x) = y$ then $x = \boxed{e^y}$ — pg 423
- Memorize If $x = \boxed{e^y}$ then $\ln(x) = y$
- Memorize $\ln(e^x) = \boxed{x}$
- Memorize If $x > 0$, then $e^{(\ln x)} = \boxed{x}$
- Memorize $\ln(e) = \boxed{1}$

Recall $\ln(x) = \log_e(x)$ — pg 422

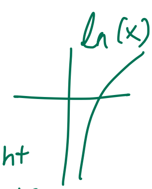
- $\ln(xy) = \boxed{\ln(x) + \ln(y)}$
- Memorize $\ln(x^r) = \boxed{r \ln(x)}$
- True or false? If x and r are positive, then $[\ln(x)]^r$ is equal to $r \ln(x)$

Answer: False. Counter example: let $r = 2$, $x = e$. Then $[\ln(e)]^2 = 1^2 = 1$, but $2 \ln(e) = 2 \cdot 1 = 2$

- If $\ln(x) = 5$, find x — $x = e^{\ln x} = \boxed{e^5}$ — pg 424
- If $e^{5-3x} = 10$, find x — $5 - 3x = \ln 10 \Rightarrow \boxed{\frac{5 - \ln 10}{3}} = x$

memorize / memorize — pg 425

- $\lim_{x \to 0^+} \ln x = \boxed{-\infty}$ and $\lim_{x \to \infty} \ln x = \boxed{\infty}$
- Sketch the graph of $y = \ln(x)$
- Sketch the graph of $y = \ln(x-2) - 1$ shift 2 right, shift 1 down



- Using l'Hospital's Rule (Sec 6.8), we can compute
  - $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} = 0$
  - $\lim_{x \to \infty} \frac{\ln x}{x^p} = \boxed{0}$ and $\lim_{x \to \infty} \frac{x^p}{\ln x} = \infty$ IF $p > 1$

This means $\ln(x)$ grows more slowly than x^p for any positive p.

Sec 6.4 Review

Look up solutions on ...

- **Memorize** $\frac{d}{dx}(\ln(x)) = \boxed{\frac{1}{x}}$ $\frac{d}{dx}(\ln|x|) = \boxed{\frac{1}{x}}$ pg 428 pg 431

- If $b > 0$, then $\frac{d}{dx}(b^x) = \boxed{(\ln b)\, b^x}$ since $b^x = e^{(\ln b)x}$ pg 433

- Differentiate $y = \ln(\sin x)$ $\frac{dy}{dx} = \frac{1}{(\sin x)}(\cos x)$ pg 429
- Find $\frac{d}{dx}\sqrt{\ln(x)} = \frac{d}{dx}[\ln(x)]^{\frac{1}{2}} = \frac{1}{2}[\ln x]^{-\frac{1}{2}}\frac{1}{x} = \frac{1}{2}\frac{1}{x}\frac{1}{\sqrt{\ln x}}$

- **MEMORIZE** $\int \frac{1}{x}\,dx = \boxed{\ln|x| + C}$ (include the absolute value sign) pg 431

- Compute $\int_1^e \frac{\ln(x)}{x}\,dx$. pg 432

$u = \ln x$
$du = \frac{1}{x}\,dx$

$\int_{\ln(1)=0}^{\ln(e)=1} u\,du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2} - 0$

Differentiate $y = (1+\sqrt{x})^x$ using Logarithmic Differentiation method

(i.e. Take ln of both sides then do implicit differentiation)

$y = (1+\sqrt{x})^x$

① Apply ln to both sides:

$\ln y = \ln\left[(1+\sqrt{x})^x\right]$

$= x \ln(1+\sqrt{x})$

② Perform implicit differentiation:

$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(x \ln(1+\sqrt{x})\right)$

$\frac{1}{y}\frac{dy}{dx} = x \frac{1}{1+\sqrt{x}}\left(\frac{1}{2}\frac{1}{\sqrt{x}}\right) + 1 \cdot \ln(1+\sqrt{x})$

$= \frac{\sqrt{x}}{2}\frac{1}{1+\sqrt{x}} + \ln(1+\sqrt{x})$

$\frac{dy}{dx} = y\left[\frac{\sqrt{x}}{2}\frac{1}{1+\sqrt{x}} + \ln(1+\sqrt{x})\right]$

$$\boxed{\frac{dy}{dx} = (1+\sqrt{x})^x\left[\frac{\sqrt{x}}{2}\frac{1}{1+\sqrt{x}} + \ln(1+\sqrt{x})\right]}$$

Sec 6.6 Review

Look up solutions on ...

$\frac{d}{dx}(\sin x) = \boxed{\cos x}$, $\frac{d}{dx}(\cos x) = \boxed{-\sin x}$, $\frac{d}{dx}(\tan x) = \boxed{(\sec x)^2}$ — pg 147–148 (Sec 2.4)

- Is $\sin^{-1} x$ the same as $\frac{1}{\sin x}$? Answer: NO — pg 474

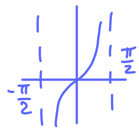
- The domain of $\arctan(x)$ is $\boxed{\text{all real numbers}}$ — pg 477

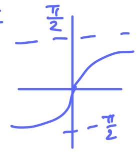
  $\underbrace{\text{domain}}_{\text{all possible inputs}}$

- The image (range) of $\arctan(x)$ is $\boxed{\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$

  $\underbrace{\text{image}}_{\text{all possible outputs}}$

- Sketch the graph of $y = \tan(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and the graph of $y = \arctan(x)$





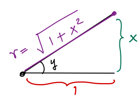
- $\lim_{x \to -\infty} \arctan(x) = \boxed{-\frac{\pi}{2}}$ $\quad$ $\lim_{x \to \infty} \arctan(x) = \boxed{\frac{\pi}{2}}$ — pg 478

- Use implicit differentiation to compute $\frac{d}{dx} \arctan(x)$ — Lecture notes 6.6

Express $\frac{dy}{dx}$ in terms of x

Let $y = \arctan(x)$

$\tan(y) = x$



Implicit Differentiation

$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x)$ $\quad$ Recall $\frac{d}{d\theta} \tan\theta = (\sec\theta)^2$

$(\sec y)^2 \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{(\sec y)^2}$

$\overset{\Delta}{=} (\cos y)^2$

$\cos y = \boxed{\frac{\text{adj}}{r} = \frac{1}{\sqrt{1+x^2}}}$

$(\cos y)^2 \overset{(*)}{=} \boxed{\frac{1}{1+x^2}}$

$\frac{dy}{dx} \overset{\Delta}{=} (\cos y)^2$

$\overset{(*)}{=} \boxed{\frac{1}{1+x^2}}$

$$\boxed{\frac{d}{dx}\left[\arctan(x)\right] = \frac{1}{1+x^2}}$$

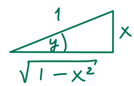
Sec 6.6 Review

Look up solutions on...

Pg 474

- Compute $\arcsin\left(\frac{1}{2}\right)$ (Don't approximate)
- Compute $\tan\left(\arcsin\left(\frac{1}{3}\right)\right)$ by drawing a triangle (Don't approximate)

- Use implicit differentiation to compute $\frac{d}{dx}(\arcsin(x))$ pg 475

Note: To get $\cos y = \frac{1}{\sqrt{1-x^2}}$, write $\sin(y) = x \Rightarrow$ 

- Use implicit differentiation to compute $\frac{d}{dx} \arccos(x)$. Lecture notes 6.6

Draw a triangle to compute $-\frac{1}{\sin(y)}$

Sec 6.8 Review

If $\lim\limits_{x\to a} f(x)=0$ and $\lim\limits_{x\to a} g(x)=0$, then $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ is an indeterminate form of type "$\frac{0}{0}$"

If $\lim\limits_{x\to a} f(x)=\infty$ (or $-\infty$) and $\lim\limits_{x\to a} g(x)=\infty$ (or $-\infty$), then $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ is an indeterminate form of type "$\frac{\infty}{\infty}$"

you can replace "$x\to a$" with "$x\to a^+$" or "$x\to a^-$" or "$x\to\infty$" or "$x\to-\infty$"

l'Hospital's Rule: (Memorize) Pg 492

Suppose f' and g' exists and $g'(x)\neq 0$.

If $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ is an indeterminate form of type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$",

then $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \boxed{\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}}$ IF ... $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ exists or is $+\infty$ or $-\infty$

Evaluate $\lim\limits_{x\to\infty}\left(1+\frac{3}{2x}\right)^{5x}$

$\lim\limits_{x\to\infty} 1+\frac{3}{2x} = 1$

$\lim\limits_{x\to\infty} 5x = \infty$

type "1^∞"

Answer

Write $y=\left(1+\frac{3}{2x}\right)^{5x}$

$$\ln y = \ln\left[\left(1+\frac{3}{2x}\right)^{5x}\right] = 5x\ln\left(1+\frac{3}{2x}\right)$$

Compute $\lim\limits_{x\to\infty}\ln y$ using l'Hospital's Rule

Write $\ln y = \dfrac{5\ln\left(1+\frac{3}{2x}\right)}{\left(\frac{1}{x}\right)}$ OR $\ln y = 5\dfrac{x}{\left[\dfrac{1}{\ln\left(1+\frac{3}{2x}\right)}\right]}$

$\underbrace{}$ looks easier

$\lim\limits_{x\to\infty} 5\ln\left(1+\frac{3}{2x}\right) = \ln(1) = 0$

$\lim\limits_{x\to\infty}\frac{1}{x} = 0$

So we can attempt l'H Rule "$\frac{0}{0}$"

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} 5\,\frac{\ln\left(1+\frac{3}{2x}\right)}{\left(\frac{1}{x}\right)} = \lim_{x\to\infty} 5\,\frac{\frac{1}{1+\left(\frac{3}{2x}\right)}\,\frac{3}{2}\left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = \lim_{x\to\infty} 5\,\frac{3}{2}\,\frac{1}{1+\left(\frac{3}{2x}\right)} = 5\,\frac{3}{2}\,\frac{1}{1}$$

by l'H Rule "$\frac{0}{0}$"

Compute $\lim\limits_{x\to\infty} y = e^{\lim\limits_{x\to\infty}(\ln y)} = \boxed{e^{\frac{15}{2}}}$

Sec 6.8 Review (Cont'd)

Look up solutions on ...

- Is $\lim_{x\to 1} \frac{\ln x}{x-1}$ an indeterminate form? If so, which type? Find $\lim_{x\to 1} \frac{\ln x}{x-1}$ pg 493
- Is $\lim_{x\to\infty} \frac{e^x}{x^2}$ an indeterminate form? If so, which type? Calculate $\lim_{x\to\infty} \frac{e^x}{x^2}$

- Is $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$ an indeterminate form? If so, which type? Evaluate $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$ pg 494

- Is $\lim_{x\to\pi^-} \frac{\sin x}{1-\cos(x)}$ an indeterminate form? If so, which type? Find $\lim_{x\to\pi^-} \frac{\sin x}{1-\cos(x)}$ pg 495
- What kind of limit is called an indeterminate form of type "$0\cdot\infty$"?
  What should you do to turn this into type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$"?
- Is $\lim_{x\to 0^+} x \ln x$ an indeterminate form? If so, which type? Find $\lim_{x\to 0^+} x \ln x$.

- What kind of limit is called an indeterminate form of type "$\infty-\infty$"? pg 496
  What should you do to turn this into type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$"?
- Is $\lim_{x\to(\frac{\pi}{2})^-} \sec x - \tan x$ an indeterminate form? If so, which type? Find $\lim_{x\to(\frac{\pi}{2})^-} \sec x - \tan x$

Indeterminate Powers Review

pg 497

- What kind of limit is called an indeterminate form of type "0^0"?
- What kind of limit is called an indeterminate form of type "∞^0"?
- What kind of limit is called an indeterminate form of type "1^∞"?

Strategy: $y = (f(x))^{g(x)}$

$$\ln(y) = \ln\left[(f(x))^{g(x)}\right] = (g(x)) \ln[f(x)]$$

Calculate $\lim_{x\to a} (\ln(y))$

Then $\lim_{x\to a} y = \lim_{x\to a} e^{(\ln(y))} = e^{\lim_{x\to a}(\ln(y))}$.

- Is $\lim_{x\to 0^+} (1+\sin(4x))^{\cot x}$ an indeterminate form? If so, which type? Find $\lim_{x\to 0^+} (1+\sin(4x))^{\cot x}$
- Is $\lim_{x\to 0^+} x^x$ an indeterminate form? If so, which type? Find $\lim_{x\to 0^+} x^x$

- Is $\lim_{x\to 0^+} (1+x)^{1/x}$ an indeterminate form? Evaluate $\lim_{x\to 0^+} (1+x)^{1/x}$.

lecture notes 6.8