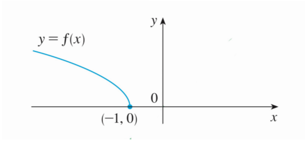
Sec 6.1 Review

1. Sketch the inverse of the graph



or answer "no inverse, since $f(x)$ is not one-to-one"

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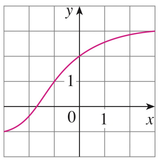
2. The graph of f is given.

(a) Why is f one-to-one?
(b) What are the domain and range of f^{-1}?
(c) What is the value of $f^{-1}(2)$?
(d) Estimate the value of $f^{-1}(0)$.



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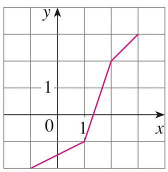
3. Sketch the inverse of the graph



or answer "no inverse"

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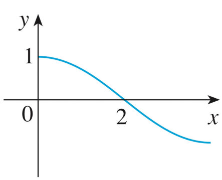
4. Sketch the inverse of the graph



or answer "no inverse"

Sec 6.1 Review

Look up solutions on ...

- A function f is called one-to-one if ...

- Is $f(x) = x^3$ with domain all real numbers one-to-one? pg 401

- Is $f(x) = x^2$ with domain all real numbers one-to-one?

- If f is one-to-one with $\underbrace{\text{domain}}_{\text{allowed inputs}}$ A and $\underbrace{\text{image/ range}}_{\text{possible outputs}}$ B,

  what is the domain of the inverse function f^{-1} of f?
  what is the image/range of the inverse function f^{-1} of f?

- If $f^{-1}(x) = y$, then $f(y) = \boxed{}$ pg 402
- If x is in the domain of f, then $f^{-1}(f(x)) = \boxed{}$
- If x is in the domain of f^{-1}, then $f(f^{-1}(x)) = \boxed{}$

Sec 6.2 Review

Look up solutions on...

- If $b > 1$, then $\lim_{x \to -\infty} b^x = \boxed{}$ and $\lim_{x \to \infty} b^x = \boxed{}$ pg 410

  Sketch the graph $y = b^x$

- If $0 < b < 1$, then $\lim_{x \to -\infty} b^x = \boxed{}$ and $\lim_{x \to \infty} b^x = \boxed{}$

  Sketch the graph $y = b^x$

- If $b > 0$, $b^{x+y} = \boxed{}$ and $(b^x)^y = \boxed{}$
- If $a > 0$, $b > 0$, $(ab)^x = \boxed{}$
- Evaluate $\lim_{x \to \infty} \left[\left(\frac{1}{2}\right)^x - 1 \right]$

- **MEMORIZE** $\frac{d}{dx}(e^x) = \boxed{}$ pg 414

- $\lim_{x \to -\infty} e^x = \boxed{}$ (Remember) and $\lim_{x \to \infty} e^x = \boxed{}$ (Remember) pg 416

- Memorize $\int e^x \, dx = \boxed{}$ pg 417

- Evaluate $\int x^2 e^{(x^3)} \, dx$.

Sec 6.3 Review

Look up solutions on...

- Memorize If $\ln(x) = y$ then $x = \boxed{}$ pg 423
- Memorize If $x = \boxed{}$ then $\ln(x) = y$
- Memorize $\ln(e^x) = \boxed{}$
- Memorize If $x > 0$, then $e^{(\ln x)} = \boxed{}$
- Memorize $\ln(e) = \boxed{}$

Recall $\ln(x) = \log_e(x)$ pg 422

- $\ln(xy) = \boxed{}$
- Memorize $\ln(x^r) = \boxed{}$
- True or false? If x and r are positive, then $[\ln(x)]^r$ is equal to $r\ln(x)$

Answer: False. Counter example: let $r = 2$, $x = e$. Then $[\ln(e)]^2 = 1^2 = 1$, but $2\ln(e) = 2.1 = 2$

- If $\ln(x) = 5$, find x pg 424
- If $e^{5-3x} = 10$, find x

memorize memorize pg 425

- $\lim\limits_{x\to 0^+} \ln x = \boxed{}$ and $\lim\limits_{x\to\infty} \ln x = \boxed{}$
- Sketch the graph of $y = \ln(x)$
- Sketch the graph of $y = \ln(x-2) - 1$
- Using l'Hospital's Rule (Sec 6.8), we can compute
  - $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt{x}} = 0$
  - $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^p} = \boxed{}$ and $\lim\limits_{x\to\infty} \dfrac{x^p}{\ln x} = \infty$ IF $p > 1$

This means $\ln(x)$ grows more slowly than x^p for any positive p.

Sec 6.4 Review

Look up solutions on...

- Memorize $\frac{d}{dx}\left(\ln(x)\right) = \boxed{\quad}$ $\frac{d}{dx}\left(\ln|x|\right) = \boxed{\quad}$ pg 428 pg 431
- If $b > 0$, then $\frac{d}{dx}\left(b^x\right) = \boxed{\quad}$ since $b^x = e^{(\ln b)x}$ pg 433

- Differentiate $y = \ln(\sin x)$ $\frac{dy}{dx} = \frac{1}{(\sin x)}(\cos x)$ pg 429
  - Find $\frac{d}{dx}\sqrt{\ln(x)} = \frac{d}{dx}\left[\ln(x)\right]^{\frac{1}{2}} = \frac{1}{2}\left[\ln x\right]^{-\frac{1}{2}} \frac{1}{x} = \frac{1}{2}\,\frac{1}{x}\,\frac{1}{\sqrt{\ln x}}$

- **MEMORIZE** $\int \frac{1}{x}\,dx = \boxed{\quad}$ (include the absolute value sign) pg 431

- Compute $\int_1^e \frac{\ln(x)}{x}\,dx$. pg 432

Differentiate $y = \left(1 + \sqrt{x}\right)^x$ using Logarithmic Differentiation method

(i.e. Take $\ln$ of both sides then do implicit differentiation)

Sec 6.6 Review

Look up solutions on ...

$\frac{d}{dx}(\sin x) = \boxed{\quad}$, $\frac{d}{dx}(\cos x) = \boxed{\quad}$, $\frac{d}{dx}(\tan x) = \boxed{\quad}$ 　 pg 147 - 148 (Sec 2.4)

- Is $\sin^{-1} x$ the same as $\frac{1}{\sin x}$? 　 Answer: NO 　 Pg 474

- The $\underbrace{\text{domain}}_{\text{all possible inputs}}$ of arctan(x) is $\boxed{\quad}$ 　 Pg 477
- The $\underbrace{\text{image}}_{\text{all possible outputs}}$ (range) of arctan(x) is $\boxed{\quad}$
- Sketch the graph of $y = \tan(x)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and the graph of $y = \arctan(x)$

- $\lim_{x \to -\infty} \arctan(x) = \boxed{\quad}$ 　 $\lim_{x \to \infty} \arctan(x) = \boxed{\quad}$ 　 Pg 478

- Use implicit differentiation to compute $\frac{d}{dx} \arctan(x)$ 　 Lecture notes 6.6

Express $\frac{dy}{dx}$ in terms of x

Sec 6.6 Review

Look up solutions on...

Pg 474

- Compute $\arcsin\left(\frac{1}{2}\right)$ (Don't approximate)
- Compute $\tan\left(\arcsin\left(\frac{1}{3}\right)\right)$ by drawing a triangle (Don't approximate)

- Use implicit differentiation to compute $\frac{d}{dx}\left(\arcsin(x)\right)$ pg 475

- Use implicit differentiation to compute $\frac{d}{dx}\arccos(x)$. Draw a triangle to compute $-\frac{1}{\sin(y)}$ Lecture notes 6.6

Sec 6.8 Review

If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type "$\frac{0}{0}$"

If $\lim_{x\to a} f(x) = \infty$ (or $-\infty$) and $\lim_{x\to a} g(x) = \infty$ (or $-\infty$), then $\lim_{x\to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type "$\frac{\infty}{\infty}$"

you can replace "$x\to a$" with "$x\to a^+$" or "$x\to a^-$" or "$x\to \infty$" or "$x\to -\infty$"

l'Hospital's Rule: (Memorize)

Pg 492

Suppose f' and g' exists and $g'(x) \neq 0$.

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ is an indeterminate form of type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$",

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \boxed{\lim_{x\to a} \frac{f'(x)}{g'(x)}}$ IF ... $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists or is $+\infty$ or $-\infty$

Evaluate $\lim_{x\to \infty} \left(1 + \frac{3}{2x}\right)^{5x}$

Sec 6.8 Review (Cont'd)

Look up solutions on ...

- Is $\lim\limits_{x\to 1} \frac{\ln x}{x-1}$ an indeterminate form? If so, which type? Find $\lim\limits_{x\to 1} \frac{\ln x}{x-1}$ pg 493
- Is $\lim\limits_{x\to\infty} \frac{e^x}{x^2}$ an indeterminate form? If so, which type? Calculate $\lim\limits_{x\to\infty} \frac{e^x}{x^2}$

- Is $\lim\limits_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$ an indeterminate form? If so, which type? Evaluate $\lim\limits_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$ pg 494

- Is $\lim\limits_{x\to\pi^-} \frac{\sin x}{1-\cos(x)}$ an indeterminate form? If so, which type? Find $\lim\limits_{x\to\pi^-} \frac{\sin x}{1-\cos(x)}$ pg 495
- What kind of limit is called an indeterminate form of type "$0\cdot\infty$"?
  What should you do to turn this into type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$"?
- Is $\lim\limits_{x\to 0^+} x\ln x$ an indeterminate form? If so, which type? Find $\lim\limits_{x\to 0^+} x\ln x$.

- What kind of limit is called an indeterminate form of type "$\infty-\infty$"? pg 496
  What should you do to turn this into type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$"?
- Is $\lim\limits_{x\to(\frac{\pi}{2})^-} \sec x - \tan x$ an indeterminate form? If so, which type? Find $\lim\limits_{x\to(\frac{\pi}{2})^-} \sec x - \tan x$

Indeterminate Powers Review

pg 497

- What kind of limit is called an indeterminate form of type "0^0"?
- What kind of limit is called an indeterminate form of type "∞^0"?
- What kind of limit is called an indeterminate form of type "1^∞"?

Strategy: $y = (f(x))^{g(x)}$

$$\ln(y) = \ln\left[(f(x))^{g(x)}\right] = (g(x))\ln[f(x)]$$

Calculate $\lim\limits_{x\to a}(\ln(y))$

Then $\lim\limits_{x\to a} y = \lim\limits_{x\to a} e^{(\ln(y))} = e^{\lim\limits_{x\to a}(\ln(y))}$.

- Is $\lim\limits_{x\to 0^+}(1+\sin(4x))^{\cot x}$ an indeterminate form? If so, which type? Find $\lim\limits_{x\to 0^+}(1+\sin(4x))^{\cot x}$
- Is $\lim\limits_{x\to 0^+} x^x$ an indeterminate form? If so, which type? Find $\lim\limits_{x\to 0^+} x^x$

- Is $\lim\limits_{x\to 0^+}(1+x)^{1/x}$ an indeterminate form? Evaluate $\lim\limits_{x\to 0^+}(1+x)^{1/x}$.

lecture notes 6.8