Sec 11.1 Review part I Answer Key

Q1. Suppose $\lim_{n\to\infty} a_n = 7$. Select all true statements below.

See pg 736 Def 1

- ✓ ☐ The sequence $\{a_n\}$ converges to 7
- ✓ ☐ The terms a_n approach 7 as n becomes large
- ✓ ☐ We can make a_n as close to 7 as we like by choosing a large enough n
- false ☐ The sequence $\{a_n\}$ diverges to 7
- false ☐ The sequence $\{a_n\}$ neither converges nor diverges

  This is never correct. If a sequence doesn't converge, it is called divergent.
- ✓ ☐ $\lim_{n\to\infty} a_n$ exists

Q2. Suppose $\lim_{n\to\infty} a_n = -\infty$. Select all true statements below.

- false ☐ The sequence $\{a_n\}$ converges → (If $\{a_n\}$ converges, we would write $\lim_{n\to\infty} a_n = \underset{\text{a number}}{L}$)
- ✓ ☐ $\lim_{n\to\infty} a_n$ does not exist
- ✓ ☐ The sequence $\{a_n\}$ diverges
- false ☐ $\lim_{n\to\infty} a_n$ exists (If $\lim_{n\to\infty} a_n$ exists, we would write $\lim_{n\to\infty} a_n = \underset{\text{a number}}{L}$)
- false ☐ The sequence $\{a_n\}$ neither converges nor diverges

  This is never correct. If a sequence doesn't converge, it is called divergent.

Sec 11.1 Review part II

Give an example of a sequence which converges to $\frac{3}{6}$

Possible answers: Exercise 19, pg 744

Give an example of a sequence which converges to 1

Possible answers: Example 4, Example 11 with $r=1$, Exercise 4, 21, 29 (pg 744)

Give an example of a sequence which converges to 0

Possible answers: Example 2, Example 6, Example 8, Example 9, Example 11 with $r=\frac{1}{3}$, Example 12, Example 13

Give an example of a convergent sequence whose terms are always positive (or 0)

Possible answers: Example 1(a), Example 4, Example 12

Give an example of a convergent sequence whose terms are sometimes positive (or 0) and sometimes negative

Possible answers: Example 1(b), Example 2, Example 8, Example 11 with $r=-\frac{1}{2}$

Give an example of a divergent sequence whose terms are always positive

Possible answers: Example 5, Example 11 with $r=2$, Exercise # 24, 25

Give an example of a divergent sequence whose terms are sometimes positive and sometimes negative

Possible answers: Example 7, Exercise 39, Example 11 with $r=-2$

Find a formula for the general term a_n of the sequences

Answers:

$$\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, \ldots$$

Example 2

$$-\frac{2}{3}, \frac{3}{9}, -\frac{4}{27}, \frac{5}{81}, \ldots$$

Example 1b

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$$

Example 1a

Sec 11.2 Review Answer Key

Copy the question and solution to Example 1, pg 749

If $a_1 + a_2 + a_3 + \dots + a_N = 1 - \frac{1}{N+1}$, evaluate $\sum_{k=1}^{\infty} a_k$. Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

solution: bottom half of Example 8, pg 752

If $a_1 + a_2 + a_3 + \dots + a_N = \frac{2}{7}\left[1-(0.3)^n\right]$, evaluate $\sum_{k=1}^{\infty} a_k$. Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

solution: Example 5(c) pg 751

Write a letter to your future self (a year from now) what it means for an infinite series $a_1 + a_2 + a_3 + \dots$ to be convergent and divergent. Write in complete sentences.

Reference Def 2, pg 748

If $a_1 + a_2 + a_3 + \dots + a_N = 5\left(1 - \left(\frac{1}{2}\right)^N\right)$, evaluate $\sum_{k=1}^{\infty} a_k$. Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

Answer: $\lim_{N\to\infty} \sum_{k=1}^{N} a_k = \lim_{N\to\infty} 5 - 5\left(\frac{1}{2}\right)^n = 5$. So $\sum_{k=1}^{\infty} a_k = 5$, so it converges [see also bottom of pg 749 (Example 2)]

If $a_1 + a_2 + a_3 + \dots + a_N = -5\left(1 - 2^N\right)$, evaluate $\sum_{k=1}^{\infty} a_k$. Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

Answer: $\lim_{N\to\infty} \sum_{k=1}^{N} a_k = \lim_{N\to\infty} \underbrace{-5}_{\text{just a number}} + \overbrace{5 \cdot 2^n}^{\text{goes to } \infty} = \infty$ So $\sum_{k=1}^{\infty} a_k$ diverges [see also bottom of pg 749 (Example 2)]

What is the Test for Divergence?

Sol: Statement $\boxed{7}$, bottom of pg 753

Can the Test for Divergence be applied to $\sum_{k=1}^{\infty} \frac{k^2}{5k^2+4}$?

Sol: Example 10, pg 754

Can the Test for Divergence be applied to $\sum_{k=1}^{\infty} \frac{k}{5k^2+4}$?

Sol: Last page, Lecture notes 11.2

Assuming the pattern continues, determine if $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$ is a geometric series. If so, determine its ratio.

Sol: Example 3, Pg 750

Sec 11.3 Review part I

Info from pg 761: The Integral Test can be applied to a series $\sum_{k=1}^{\infty} a_k$ IF:

① $a_k = f(k)$ for $k = 1, 2, 3, \dots$
② f is continuous, positive, and decreasing on $[1, \infty)$.

* For the series given in Webwork # 1, 2 determine whether the Integral Test can or cannot be applied to the series

* For the series given in Webwork # 4, 6 determine whether the Integral Test can or cannot be applied to the series

* For the series given in Webwork # 7 determine whether the Integral Test can or cannot be applied to the series

Answer. The series all satisfy the conditions for the Integral Test written above. (Why?)
So, YES, the Integral Test can be applied to all of them.

Sec 11.3 Review part II Answer key

Solution for $\sum_{k=1}^{\infty} f(k)$

Step 1: (Check whether $\int_1^{\infty} f(x)\,dx$ converges)

$$\int_1^{\infty} f(x)\,dx \overset{def}{=} \lim_{t\to\infty} \int_1^{t} f(x)\,dx$$

$$= \lim_{t\to\infty} 1 - \frac{\ln(t)+1}{t}$$

$$= 1 - \lim_{t\to\infty} \frac{\ln(t)+1}{t}$$

$$= 1 - \lim_{t\to\infty} \frac{\left(\frac{1}{t}\right)}{1} \quad \left(\text{by l'Hospital's Rule } \frac{\infty}{\infty}\right)$$

$$= 1 - 0$$

Step 2: Since $\int_1^{\infty} f(x)\,dx$ converges, $\sum_{k=1}^{\infty} f(k)$ also converges by the Integral Test.

Side note: We don't know the actual value for $\sum_{k=1}^{\infty} f(k)$. We just know $\sum_{k=1}^{\infty} f(k)$ is convergent.

Solution for $\sum_{k=4}^{\infty} h(k)$

Step 1: (Check whether $\int_4^{\infty} h(x)\,dx$ converges)

$$\int_4^{\infty} h(x)\,dx \overset{def}{=} \lim_{t\to\infty} \int_4^{t} h(x)\,dx$$

$$= \lim_{t\to\infty} \ln(\ln t) - \ln(\ln 4)$$

$$= \infty \quad \text{since } \lim_{t\to\infty} \ln(\ln t) = \infty \text{ and } \ln(\ln 4) \text{ is a number}$$

Step 2: Since $\int_4^{\infty} h(x)\,dx$ diverges,

$\sum_{k=4}^{\infty} h(k)$ also diverges by the Integral Test.

Side note:

Since $h(k)$ is positive for all $k = 4, 5, 6, \ldots$ we write $\sum_{k=4}^{\infty} h(k) = \infty$