Sec 11.1 Review part I

Q1. Suppose $\lim_{n\to\infty} a_n = 7$. Select all true statements below.

- ☐ The sequence $\{a_n\}$ converges to 7
- ☐ The terms a_n approach 7 as n becomes large
- ☐ We can make a_n as close to 7 as we like by choosing a large enough n
- ☐ The sequence $\{a_n\}$ diverges to 7
- ☐ The sequence $\{a_n\}$ neither converges nor diverges
- ☐ $\lim_{n\to\infty} a_n$ exists

Q2. Suppose $\lim_{n\to\infty} a_n = -\infty$. Select all true statements below.

- ☐ The sequence $\{a_n\}$ converges
- ☐ $\lim_{n\to\infty} a_n$ does not exist
- ☐ The sequence $\{a_n\}$ diverges
- ☐ $\lim_{n\to\infty} a_n$ exists
- ☐ The sequence $\{a_n\}$ neither converges nor diverges

Sec 11.1 Review part II

Give an example of a sequence which converges to $\frac{3}{6}$

Possible answers: Exercise 19, pg 744

Give an example of a sequence which converges to 1

Possible answers: Example 4, Example 11 with $r = 1$, Exercise 4, 21, 29 (pg 744)

Give an example of a sequence which converges to 0

Possible answers: Example 2, Example 6, Example 8, Example 9, Example 11 with $r = \frac{1}{3}$, Example 12, Example 13

Give an example of a convergent sequence whose terms are always positive (or 0)

Possible answers: Example 1(a), Example 4, Example 12

Give an example of a convergent sequence whose terms are sometimes positive (or 0) and sometimes negative

Possible answers: Example 1(b), Example 2, Example 8, Example 11 with $r = -\frac{1}{2}$

Give an example of a divergent sequence whose terms are always positive

Possible answers: Example 5, Example 11 with $r = 2$, Exercise # 24, 25

Give an example of a divergent sequence whose terms are sometimes positive and sometimes negative

Possible answers: Example 7, Exercise 39, Example 11 with $r = -2$

Find a formula for the general term a_n of the sequences

Answers:

$$\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, \ldots$$

Example 2

$$-\frac{2}{3}, \frac{3}{9}, -\frac{4}{27}, \frac{5}{81}, \ldots$$

Example 1b

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$$

Example 1a

Sec 11.2 Review

Copy the question and solution to Example 1, pg 749

If $a_1 + a_2 + a_3 + \dots + a_N = 1 - \frac{1}{N+1}$, evaluate $\sum_{k=1}^{\infty} a_k$.

Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

If $a_1 + a_2 + a_3 + \dots + a_N = \frac{2}{7}\left[1 - (0.3)^n\right]$, evaluate $\sum_{k=1}^{\infty} a_k$.

Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

Write a letter to your future self (a year from now) what it means for an infinite series $a_1 + a_2 + a_3 + \dots$. to be convergent and divergent. Write in complete sentences.

Reference Def 2, pg 748

If $a_1 + a_2 + a_3 + \dots + a_N = 5\left(1 - \left(\frac{1}{2}\right)^N\right)$, evaluate $\sum_{k=1}^{\infty} a_k$.

Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

If $a_1 + a_2 + a_3 + \dots + a_N = -5\left(1 - 2^N\right)$, evaluate $\sum_{k=1}^{\infty} a_k$.

Determine whether $\sum_{k=1}^{\infty} a_k$ converges.

What is the Test for Divergence?

Can the Test for Divergence be applied to $\sum_{k=1}^{\infty} \frac{k^2}{5k^2+4}$?

Can the Test for Divergence be applied to $\sum_{k=1}^{\infty} \frac{k}{5k^2+4}$?

Assuming the pattern continues, determine if $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots$ is a geometric series. If so, determine its ratio.

Sec 11.3 Review part I

Info from pg 761: The Integral Test can be applied to a series $\sum_{k=1}^{\infty} a_k$ IF:

① $a_k = f(k)$ for $k = 1, 2, 3, \ldots$

② f is continuous, positive, and decreasing on $[1, \infty)$.

* For the series given in Webwork # 1, 2 determine whether the Integral Test can or cannot be applied to the series

* For the series given in Webwork # 4, 6 determine whether the Integral Test can or cannot be applied to the series

* For the series given in Webwork # 7 determine whether the Integral Test can or cannot be applied to the series

Sec 11.3 Review part II

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* Suppose we know that, for a mystery function f,
  - f is continuous, positive, and decreasing on $[1, \infty)$
  - If $t > 1$, then $\int_1^t f(x)\,dx = 1 - \frac{\ln(t)+1}{t}$

Use the Integral Test to determine whether $\sum_{k=1}^{\infty} f(k)$ converges or diverges.

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* Suppose we know that, for a mystery function h,
  - h is continuous, positive, and decreasing on $[4, \infty)$
  - If $t > 4$, then $\int_4^t h(x)\,dx = \ln(\ln t) - \ln(\ln 4)$

Use the Integral Test to determine whether $\sum_{k=4}^{\infty} h(k)$ converges or diverges.

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* Complete the two problems from Lec 11.3 pg 3 without looking at the solution, then again looking at the solution.