Name: ____

Math 1152Q: Fall 2018
Week 7 Quiz sample

Summary:

- Integration by parts
- apply partial fraction decomposition to rational function (with denominators that have deg 2 or 3);
- determine whether an improper integral diverges or converges; evaluate the convergent integral;

1. (Optional, for an extra token) Write 10 or more distinct (correctly spelled) preferred (first) names of students who are present during the quiz (whose name is not the same as your name) and be able to match names to faces.

   Example: 'write the first names of the students who are currently sitting in the back row'.

1 Sec 7.1 only, integration by parts

2. (Integration by parts)
   1.) Evaluate $\int \ln\left(x+\sqrt{1+x^2}\right)$ dx or 2.) $\int x \ \tan^2 x$ dx or 3.) $\int \cos(\sqrt{x})$ dx or
   4.) Evaluate $\int x^2 (\ln x)^2$ dx or 5.) omitted or 6.) $\int \cos(\ln x)$ dx or
   7a.) How can you derive the formula for Integration by Parts? or
   7b.) Evaluate $\int_0^{\frac{\pi}{2}} x \ \cos(2x)$ dx or
   7c.) Suppose $f(1)$=2, $f(4)$=7, $f'(1) = 5$, $f'(4)$=3. Suppose f'' is continuous. Evaluate $\int_1^4 x \ f''(x)$ dx or 7d.) Evaluate $\int \arctan x$ dx or 7e.) $\int e^x \cos x$ dx or
   7f.) A particle that moves along a straight line has velocity $v(t) = t^3 e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

3. (a) Evaluate $\int (x+2)\sin(3x)$ dx
   (b) Prove that your previous answer is correct by differentiating and applying the Fundamental Theorem of Calculus.

2 Sec 7.4 only, integrals of rational functions

4. (Integrating rational functions)
   (a) Evaluate $\int \frac{2x+1}{x^2-4}\,dx$. Although it is possible to evaluate this using the trig substitution method, please evaluate by first doing partial fraction decomposition method.
   (b) Evaluate $\int \frac{10}{(x+5)(x-2)}$ dx.
   (c) Provide a computation that is either a formal verification (that is, a proof) or simply a sanity-check for your answer to the previous question.
   (d) Evaluate $\int \frac{9}{(x-6)(x+3)}$ dx or $\int \frac{12}{(x-2)(x+1)}$ dx or $\int \frac{8}{(x-1)(x+3)}$ dx.
   (e) Evaluate $\int \frac{x+4}{x^2+2x+5}$ dx

3 7.8 Proper and improper Integrals and evaluating them using Sec 7.1, 7.4, and u-substitution

5. (Sec 7.8 improper integrals)
   (a) Find the values of p for which the integral $\int_e^\infty \frac{6}{x(\ln x)^p}$ dx converges. Evaluate the integral for these values of p.
   (Hint: u-substitution. Check what happens when $p = 1$, when $p < 1$, and when $p > 1$.)
   (b) Determine whether
   $$\int_2^\infty \left(\frac{1}{e^5}\right)^x \text{ dx}$$
   is convergent or divergent. If it is convergent, evaluate it.
   (c) Determine whether
   $$\int_2^\infty \frac{1}{x^2 + 8x - 9} \text{ dx}$$
   is convergent or divergent. If it is convergent, evaluate it.
   (d) Determine whether
   $$\int_0^1 \frac{4}{x^5} \text{ dx}$$
   is convergent or divergent. If it is convergent, evaluate it.
   (e) Determine whether
   $$\int_0^1 \frac{4}{x^{0.5}} \text{ dx}$$
   is convergent or divergent. If it is convergent, evaluate it.
   (f) Determine whether
   $$\int_2^3 \frac{2}{\sqrt{3-x}} \text{ dx}$$
   is convergent or divergent. If it is convergent, evaluate it.
   (g) Evaluate the integral
   $$\int_1^\infty \frac{3}{x^6} \text{ dx.}$$
   (h) Evaluate the integral
   $$\int_1^\infty \frac{1}{(4x+2)^3} \text{ dx.}$$
   (i) Evaluate the integral
   $$\int_1^\infty \frac{1}{\sqrt{x+9}} \text{ dx.}$$
   (j) Evaluate the integral
   $$\int_1^\infty x\ e^{-9x} \text{ dx.}$$
   (k) Evaluate the integral
   $$\int_1^\infty x\ e^{-9x} \text{ dx}$$

6. (Various definite and indefinite integrals).

(a) Is $\int_1^2 t^3 \ln(t)dt$ a proper or improper integral?

(b) Evaluate $\int t^3 \ln(t)dt$ and $\int_1^2 t^3 \ln(t)dt$

(c) Perform either a proof or a reality check for the previous problem. For example, differentiate your answer (for a proof) or check that your definite integral is a positive number, since $t^3 ln(t)$ is positive for $t > 1$.

(d) Is $\int_0^1 x\ e^{-x^2}$ a proper or improper integral?

(e) Evaluate $\int_0^1 x\ e^{-x^2}$

(f) Perform a reality check for your answer to the previous problem.

(g) Evaluate $\int e^{\sqrt{x}}$ dx

(h) Prove that your previous answer is correct by differentiating and applying the Fundamental Theorem of Calculus.

(i) Is $\int_0^9 \frac{1}{\sqrt{x}}$ dx an improper integral?

(j) Determine whether $\int_0^{25} \frac{1}{\sqrt{x}}$ dx or $\int_0^{16} \frac{1}{\sqrt{x}}$ dx or $\int_0^9 \frac{1}{\sqrt{x}}$ dx is convergent or divergent. If it is convergent, evaluate the integral.

7. (From class handouts)

(a) Is the integral $\int_0^{\pi} \sin^3(5x)\, dx$ a proper or improper integral?

(b) Without explicitly trying to evaluate this integral, determine whether it is possible for $\int_0^{\infty} x^2 e^{-x}\, dx$ to be convergent to a negative value.

(c) Is $\int_0^{\infty} x^2 e^{-x}\, dx$ convergent or divergent? If convergent, evaluate it.

(d) Evaluate 1.) $\int_0^{\infty} e^{-2x}$ dx or 2.) $\int_1^{\infty} \frac{1}{\sqrt{x}}$ or 3.) $\int_1^{\infty} \sin^2 x$ dx or 4.) $\int_1^{\infty} \frac{1}{x^2+2x-3}$ dx.

(e) 1.) Evaluate $\int_0^8 \frac{1}{\sqrt[3]{x}}$ dx or 2.) Evaluate $\int_1^{\frac{\pi}{2}} \sec x$ or 3.) Evaluate $\int_0^5 \frac{x}{x-2}$ dx or 4.) Use the Comparison Theorem to determine whether $\int_1^{\infty} \frac{x}{x^3+1}$ dx converges.

8. (Sec 7.8 Improper integrals)

(a) When is an integral improper? Hint: There are two kinds. Copy definitions from Sec 7.8 pg 527 and 531.

(b) When is an integral proper? (The answer is when it is not improper, but explain what needs to happen for a definite integral to be proper).

(c) True or False, and why? Let $f(x)$ be continuous everywhere. $\int_a^{\infty} f(x)$ dx $= \lim_{t\to\infty} \int_a^t f(x)$ dx **T** **F**

Justification:

(d) True or False, and why? Let $f(x)$ be continuous everywhere. $\int_a^{\infty} f(x)$ dx $= \lim_{a\to\infty} \int_a^t f(x)$ dx **T** **F**

Justification:

(e) True or False, and why? The integral $\int_2^3 \sqrt{x-2}$ dx is improper. **T** **F**

(f) True or False, and why? $\int_0^1 \frac{27}{x^5}$ dx is improper. **T** **F**

(g) Evaluate $\int_0^1 \frac{27}{x^5}$ dx

(h) True or False? $\int_{-1}^1 \frac{1}{x}$ dx is improper. **T** **F**

(i) Evaluate $\int_{-1}^1 \frac{1}{x}$ dx

(j) Determine whether $\int_0^1 9x^2 \ln(x)$ dx converges or diverges. If it converges, evaluate it.

(k) Evaluate $\int_4^8 \frac{4}{x\sqrt{x^2-16}}dx$ OR $\int_{-7}^7 \frac{1}{\sqrt{49-x^2}}dx$. If it converges, evaluate it.

(l) Determine whether $\int_e^\infty \frac{1}{x(\ln x)^3}$ dx converges or diverges. If it converges, evaluate the integral.

(m) Determine whether

$$\int_2^\infty \left(\frac{1}{e^5}\right)^x \text{ dx}$$

is convergent or divergent. If it is convergent, evaluate it.

(n) Determine whether $\int_2^\infty \frac{1}{x^2+8x-9}$ dx is convergent or divergent. If it is convergent, evaluate it.

(o) Determine whether $\int_0^1 \frac{4}{x^5}$ dx is convergent or divergent. If it is convergent, evaluate it.

(p) Determine whether $\int_0^1 \frac{4}{x^{0.5}}$ dx is convergent or divergent. If it is convergent, evaluate it.

(q) Determine whether $\int_2^3 \frac{2}{\sqrt{3-x}}$ dx is convergent or divergent. If it is convergent, evaluate it.

(r) Write 2 improper integrals (different from above) so that one is convergent and the other is divergent.

(s) Write 2 proper (definite) integrals that are different from above.

(t) Write 2 indefinite integrals.