Math 1152Q: Fall '18 **Week 2 SAMPLE Quiz, Friday, Sept 7**

Topics: Sec 11.1: Definition of convergence, ϵ, N proof, bounded sequences, decreasing/increasing sequences, Monotone Sequence Theorem.

1. (Sec 11.1 Vocabulary)
   (a) Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$? Use Def 5, with M and N. (Answer: Copy from Sec 11.1 Def 5 page 697)
   (b) Given a sequence $\{a_n\}$, what does $\lim_{n\to\infty} a_n = 4$ mean? Use the ϵ and N definition. (Answer: Copy from Sec 11.1 Def 2 pg 696)

2. (Sec 11.1 ϵ, N proof) Let ϵ be a positive number smaller than 1.
   (a) The sequence $a_n = \dfrac{5n^2-9}{n^2-4}$ converges to 5. Choose N so that $|a_n - 5| < \epsilon$ whenever $n > N$. (Answer: `https://egunawan.github.io/fall18/notes/notes11_1choosingN.pdf`)
   (b) The sequence $a_n = \dfrac{n-1}{7n+4}$ converges to $1/7$. Choose N so that $|a_n - 1/7| < \epsilon$ whenever $n > N$. Show that this N works.
   (c) Give *a* positive number N such that, $\dfrac{1}{n^2-8} < \epsilon$ for all $n > N$. Show that this N works.
   (d) The sequence $a_n = \dfrac{2n+4}{5n-8}$ converges to $2/5$. For any number $\epsilon > 0$, choose N so that if $n > N$, then $\left|\dfrac{2}{5} - a_n\right| < \epsilon$.
   (e) The sequence $a_n = \dfrac{n^2+1}{7n^2+5}$ converges to $1/7$. For any number $\epsilon > 0$, find N so that $|1/7 - a_n| < \epsilon$ as long as $n > N$.

3. (Sec 11.1 Bounded sequences, monotonic sequences)
   i.) Fill in the blanks with either the sign $\leq$ or $\geq$.

$$\frac{5n!}{2^n} \;\_\_\_\_\_\_\; \left(\frac{1}{2}\right)^n \quad \text{for all } n \geq 1$$
$$\frac{n-1}{7n+4} \;\_\_\_\_\_\_\; \frac{1}{7} \quad \text{for all } n \geq 1$$
$$\frac{n+1}{7n-4} \;\_\_\_\_\_\_\; \frac{1}{7} \quad \text{for all } n \geq 1$$

   ii.) (Graphing Review) Sketch each function. Label the asymptote/s and zero/s of the graph.
   (a) $f(x) = \dfrac{x-1}{7x+4}$
   Answer:
   - $f(x) = \dfrac{x-1}{7x+4}$ has a horizontal asymptote at $\frac{1}{7}$ because $\lim\limits_{x\to\infty} f(x) = 1/7$.
   - Note that $f(x) = \dfrac{x-1}{7x+4}$ is not defined for $x = -4/7$ and that $-4/7$ is not a zero of the numerator $x - 1$. This tells us that $f(x) = \dfrac{x-1}{7x+4}$ has a vertical asymptote at $x = -\dfrac{4}{7}$.

- Definition: The graph of $y = f(x)$ is said to have a vertical asymptote $x = a$ if $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$.
- To figure out whether your graph approaches $+\infty$ or $-\infty$ to the right, plug in a number bigger than a and estimate whether it looks very large (positive) or very large (negative).

(b) $g(x) = \dfrac{x+1}{7x-4}$

(c) $h(x) = \dfrac{1}{x+5}$

iii.) By just looking at your sketches above, determine whether each of the following sequences is increasing or decreasing (or neither) for $n = 1, 2, 3, \ldots$.

(a) $\left\{\dfrac{n-1}{7n+4}\right\}_{n=1,2,3,\ldots}$ (b) $\left\{\dfrac{n+1}{7n-4}\right\}_{n=1}^{\infty}$ (c) $\left\{\dfrac{1}{n+5}\right\}_{n=1}^{\infty}$

iv.) Use your work above to quickly give a lower bound (a number m) and an upper bound (a number M) for each of the following sequences.

(a) $\left\{\dfrac{n-1}{7n+4}\right\}_{n=1,2,3\ldots}$ (b) $\left\{\dfrac{n+1}{7n-4}\right\}_{n=1}^{\infty}$ (c) $\left\{\dfrac{1}{n+5}\right\}_{n=1}^{\infty}$

4. (Sec 11.1 monotonic sequence theorem) Recall that lower and upper bounds are not unique!

(a) True or false? The sequence

$$\left\{\frac{3n-6}{6n+2}\right\}_{n=1}^{\infty}$$

is bounded. Give numbers which are *an* upper bound and *a* lower bound (if true) or justify (if false).

(Answer: True. For example, the sequence is bounded by $-\frac{3}{8}$ and $\frac{1}{2}$. Sketch the corresponding function).

(b) True or false? The sequence

$$\left\{5ne^{-6n}\right\}_{n=1}^{\infty}$$

is bounded. Give *an* upper bound and *a* lower bound (if true) or justify (if false).

(Answer: True. The least upper bound is $\frac{5}{e^6}$ and the greatest lower bound is 0. You can compute the derivative of the corresponding function to show that this sequence is decreasing.)

(c) True or false? Every bounded sequence is convergent. Justify (if T) or give a counterexample (if F).

(d) True or false? There exists an increasing sequence that converges to 10. Provide an example (if T) or justify (if F).

(e) True or false? There exists an increasing and bounded sequence that does not converge. Provide an example (if T) or justify (if F).

(f) True or false? There is a non-monotonic sequence that converges to 4. Provide an example (if T) or justify (if F).

(g) True or false? Not every bounded sequence is convergent. Give an example of a bounded sequence that is not convergent (if T) or justify (if F).

(h) True or false? There exists a decreasing sequence that converges to 10. Provide an example (if T) or justify (if F).