TASK 1.
Go to page 493. Read/skim the introduction.
Below, write down the question and solution to Example 1 (Sec 7.4, page 493). Make sure you understand every step. (Write the long division below without looking at the book.)

Motivation

Note that

$$\frac{1}{x-1}-\frac{1}{x+3}=\frac{(x+3)-(x-1)}{(x-1)(x+3)}=\frac{4}{x^2+2x-3}$$

The purpose of partial fractions is to reverse this process.

Partial Fractions	**Common Denominator**	Rational function
$\frac{1}{x-1}-\frac{1}{x+3}$	$\rightarrow$	$\frac{4}{x^2+2x-3}$
Easy to integrate	**Partial Fraction Decomposition**	Difficult to integrate
$\int\left(\frac{1}{x-1}-\frac{1}{x+3}\right)dx$	$\leftarrow$	$\int\frac{4}{x^2+2x-3}\,dx$

Definition

Let $f(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are **polynomials**.

- $f(x)$ is called a **rational function**.
- If **the degree** of $p(x)$ is **less than the degree** of $q(x)$, then $f(x)$ is called **proper**.

Procedure

Suppose $f(x)$ is a **proper rational function**.

1. Factor the Denominator.
2. Perform Partial Fraction Decomposition.
3. Clear Denominators by

   ____.
4. Solve for Unknowns by

   ____.

TASK 2a. Case 1 – (fill in the blank, copy from Sec 7.4, pg 494)

The denominator is a product of distinct linear factors

Suppose the denominator of a **proper rational function** can be written as

$$\left(a_1x+b_1\right)\left(a_2x+b_2\right)\quad\left(a_kx+b_k\right)$$

where no factor is repeated (and no factor is a constant multiple of another). Then there exists constants A_1, A_2, , and A_k such that

____.

Task 2b. Below, write down the question and solution to Example 3 (Sec 7.4, pg 495) but change the constant "a" to your favorite positive number.

Study (or skim) the solution to Example 2 (Sec 7.4, pg 494).
(Optional) TASK 3a: Below, write down the solution to Example 2 (Sec 7.4, pg 494).

TASK 3b. Follow book's Example 2 above (or a different strategy) to solve this similar problem:

Consider the function $f(x)=\dfrac{3x^2+7x-2}{x^3-x^2-2x}$.

a. Find the partial fraction decomposition for $f(x)$.

b. Evaluate $\int f(x)\,dx$.

Verify your answer with a computer. (Type "partial fraction" on Wolfram|Alpha).

TASK 4a. Case 2 – (fill the blank by copying the line labeled 7 from Sec 7.4, top of pg 496)
The denominator is a product of linear factors, some of which are repeated

Suppose the repeated linear factor $(ax+b)^m$ appears in the denominator of a **proper rational function**. The partial fraction decomposition has a partial fraction for each power of $(ax+b)$ up to and including the m th power. That is, there exists constants A_1, A_2, , and A_m such that the partial fraction decomposition contains the sum

____.

TASK 4b. Finish the following incomplete solution:

Consider the function $f(x)=\dfrac{5x^2-3x+2}{x^3-2x^2}$.

1. Find the partial fraction decomposition for $f(x)$.

Solution: We use the above Case 2 theorem to know that we can write

$$\frac{5x^2-3x+2}{x^3-2x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-2}$$

From here, follow the same strategy of Example 4 (or another method) to compute A, B, and C.
Verify your work - type on Wolfram|Alpha: “partial fraction ((5x^2 - 3x + 2)/(x^3 - 2x^2))”

2. Evaluate $\int f(x)\,dx$.

Task 4a. Case 3 – (fill in the blank by copying from pg 497)

The denominator contains irreducible quadratic factors, none of which is repeated

Suppose the denominator of a **proper rational function** has the factor $ax^2 + bx + c$, where $b^2 - 4ac < 0$. There exists constants A and B such that the partial fraction decomposition contains a term of the form

____.

Task 4b. Write down the integral to Example 5 (pg 497). Copy down the solution only as far as computing A, B, C. Show your work.

(Optional: finish computing the integral)

Note: Case I, II, III are the most common types that will be useful for differential equations. (You can read about case IV (pg 499) if you're interested, but we won't discuss it.)