Calculus 1 review (Stewart Chapters 2-4) for Math 1152Q

Chapter 2

2.1 The tangent and velocity problems and precalculus

1. [**Q**] **True** or **False**. If $f(x) = \dfrac{x^2 - 4}{x - 2}$ and $g(x) = x + 2$, then we can say the functions f and g are equal.

2.3 Calculating limits using the limit laws

1. [**Q**] The statement "Whether or not $\lim_{x\to a} f(x)$ exists, depends on how $f(a)$ is defined," is true
   (a) sometimes
   (b) always
   (c) never

2. [**Q**] If a function f is not defined at $x = a$,
   (a) $\lim_{x\to a} f(x)$ cannot exist
   (b) $\lim_{x\to a} f(x)$ could be 0
   (c) $\lim_{x\to a} f(x)$ must approach ∞
   (d) none of the above.

3. [**Q**] If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \dfrac{f(x)}{g(x)}$
   (a) does not exist
   (b) must exist
   (c) not enough information

The following two problems to be used in a sequence:

4. **[D]** The reason that $\lim_{x\to 0} \sin(1/x)$ does not exist is:
   (a) because no matter how close x gets to 0, there are x's near 0 for which $\sin(1/x) = 1$, and some for which $\sin(1/x) = -1$
   (b) because the function values oscillate around 0
   (c) because "1/0" is undefined
   (d) all of the above

5. **[D]** $\lim_{x\to 0} x^2 \sin(1/x)$
   (a) does not exist because no matter how close x gets to 0, there are x's near 0 for which $\sin(1/x) = 1$, and some for which $\sin(1/x) = -1$
   (b) does not exist because the function values oscillate around 0
   (c) does not exist because "1/0" is undefined
   (d) equals 0
   (e) equals 1

2.4 Continuity

1. **[Q]** You know the following statement is true:

   If $f(x)$ is a polynomial, then $f(x)$ is continuous.

   Which of the following is also true?
   (a) If $f(x)$ is not continuous, then it is not a polynomial.
   (b) If $f(x)$ is continuous, then it is a polynomial.
   (c) If $f(x)$ is not a polynomial, then it is not continuous.

2.5 Limits involving infinity

1. **[Q]** **True** or **False**. Consider a function $f(x)$ with the property that $\lim_{x\to a} f(x) = 0$. Now consider another function $g(x)$ also defined near a. Then $\lim_{x\to a}[f(x)g(x)] = 0$.

2. **[Q]** **True** or **False**. If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then $\lim_{x\to a}[f(x) - g(x)] = 0$.

3. **[Q]** What is the maximum number of horizontal asymptotes that a function can have?
   (a) one
   (b) two
   (c) three
   (d) as many as we want

2.7 Derivatives

1. **[Q]** **True** or **False**. The function $f(x) = x^{1/3}$ is continuous at $x = 0$.
2. **[Q]** **True** or **False**. If $f(x) = x^{1/3}$ then $f'(0)$ exists.
3. **[P]** **True** or **False**. If $f(x) = x^{1/3}$ then there is a tangent line at $(0, 0)$.

2.8 The derivative as a function

1. **[P]** If $f'(a)$ exists, $\lim_{x \to a} f(x)$
   (a) it must exist, but there is not enough information to determine it exactly
   (b) equals $f(a)$
   (c) equals $f'(a)$
   (d) it may not exist
2. **[P]** Your mother says "If you eat your dinner, you can have dessert." You know this means, "If you don't eat your dinner, you cannot have dessert." Your calculus teacher says, "If f if differentiable at x, f is continuous at x." You know this means
   (a) if f is not continuous at x, f is not differentiable at x.
   (b) if f is not differentiable at x, f is not continuous at x.
   (c) knowing f is not continuous at x, does not give us enough information to deduce anything about whether the derivative of f exists at x.
3. **[Q]** A slow freight train chugs along a straight track. The distance it has traveled after x hours is given by a function $f(x)$. An engineer is walking along the top of the box cars at the rate of 3 mi/hr in the same direction as the train is moving. Her speed relative to the ground is
   (a) $f(x) + 3$
   (b) $f'(x) + 3$
   (c) $f(x) - 3$
   (d) $f'(x) - 3$

Chapter 3

3.1 Derivatives of polynomials and exponential functions

1. **[Q]** $\frac{d}{dx}(e^7)$ equals
   (a) $7e^6$
   (b) e^7
   (c) 0

2. **[P]** $\lim_{x\to 1} \frac{x^{10}-1}{x-1}$
   (a) does not exist, because $\frac{0}{0}$ is not defined
   (b) equals 10, because it is exactly the derivative of x^{10}, at $x = 1$
   (c) equals 1, because $(1)^9 = 1$.

3.4 Derivatives of trigonometric functions

1. **[P]** $\lim_{x\to 0} \frac{\sin x}{x} = 1$ means that
   (a) $\frac{0}{0} = 1$
   (b) the tangent to the graph of $y = \sin x$ at $(0, 0)$ is the line $y = x$
   (c) you can cancel the x's.
   (d) $\sin x = x$ for x near 0.

2. **[Q]** If $f(x) = \sin x$ then
   (a) $f(x) = f''''(x)$
   (b) $f(x) = -f''(x)$
   (c) $f'(x) = \cos(x)$
   (d) all of the above.

3. **[P]** $\lim_{h\to 0} \frac{\sin(2x+h) - \sin(2x)}{h}$ equals
   (a) $\cos x$
   (b) $\cos(2x)$
   (c) zero
   (d) does not exist because "0/0" is not defined

3.5 The Chain Rule

1. **[Q]** If f and g are both differentiable and $h = f \circ g$, $h'(2)$ equals
   (a) $f'(2) \circ g'(2)$
   (b) $f'(2)g'(2)$
   (c) $f'(g(2))g'(2)$
   (d) $f'(g(x))g'(2)$

3.7 Derivatives of logarithmic functions

1. **[Q]** **True** or **False**. $\frac{d}{dx}\ln(\pi) = \frac{1}{\pi}$.

2. **[Q]** Your calculus book says that $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$. This means:
   (a) e is not really a number because it is a limit
   (b) e cannot be computed
   (c) the sequence of numbers $\left(\frac{2}{1}\right), \left(\frac{3}{2}\right)^2, \left(\frac{4}{3}\right)^3, ..., \left(\frac{101}{100}\right)^{100}, ...$ get as close as you want to the number e

3. **[P]** When you read in the newspaper thing like inflation rate, interest rate, birth rate, etc., it always means $\frac{f'}{f}$, not f' itself.

   True or **False**. $\frac{f'}{f}$ is not the derivative of a function.

4.5 L' Hospital's Rule

1. **[Q]** Consider the functions $f(x) = e^x$ and $g(x) = x^{1000}$. As $x \to \infty$ which of the following is true?

   (a) f grows faster than g.

   (b) g grows faster than f.

   (c) We cannot determine.

   (d) They grow at the same rate like all exponentials.

2. **[Q]** The limit $\lim_{x \to \infty} (xe^{1/x} - x)$

   (a) Does not exist because $\infty - \infty$ is not defined.

   (b) Converges to 1.

   (c) Is ∞ because $xe^{1/x}$ grows faster than x.

   (d) Converges to 0.

Source: `http://pi.math.cornell.edu/~GoodQuestions/materials.html`