Name:_________________ **Representations of Functions as Power Series** Notes

A **power series** defines a **function** on its **interval of convergence**.

Task 1. Read pages 752-753. Write a brief summary (1-2 sentences).

Combining Power Series

Theorem Combining Power Series

Suppose the power series $\sum c_n x^n$ and $\sum d_n x^n$ converge absolutely to $f(x)$ and $g(x)$, respectively, on an interval I.

1. **Sum and Difference**

   The power series $\sum (c_n \pm d_n) x^n$ converges absolutely to $f(x) \pm g(x)$ on I.

2. **Multiplication by a power**

   The power series $x^m \sum c_n x^n = \sum c_n x^{n+m}$ converges absolutely to $x^m f(x)$ on I, provided m is an integer such that $k + m \geq 0$ for all terms of the series.

3. **Composition**

   If $h(x) = bx^m$, where m is a positive integer and b is a real number, the power series $\sum c_n \left[h(x) \right]^n$ converges absolutely to the composite function $f(h(x))$ for all x such that $h(x)$ is in I.

Task 2 (Copy Sec 11.9, Example 1 from page 753).
Try to specify where the above Theorem (part 3) is used. What are f (t) and h (x) ?
Give the interval of convergence of the series using above Theorem (part 3).

Task 3 (Copy Sec 11.9 Example 2, pg 753):
Use the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ for } |x| < 1$$

to find a power series representation for $\frac{1}{x+2}$.
Try to specify where the above Theorem (part 3) is used. What are f (t) and h (x) ?
Give the interval of convergence of the new series by applying the above Theorem.

Differentiating and Integrating Power Series

Theorem **Differentiating and Integrating Power Series (Sec 11.9, pg 754)**

Let the function f be defined by the power series $\sum c_n (x-a)^n$ on its interval of convergence I. THEN:

1. f is a **continuous** function on I.

2. The power series may be differentiated **term by term**.

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} c_n (x-a)^n\right] = \frac{d}{dx}\left[c_0 + \sum_{n=1}^{\infty} c_n (x-a)^n\right] = \sum_{n=1}^{\infty} c_n n (x-a)^{n-1}.$$

   The resulting power series converges to $f'(x)$ at all points in the interior of I.

3. The power series may be integrated **term by term**.

$$\int\left[\sum_{n=0}^{\infty} c_n (x-a)^n\right] dx = \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1} + C \text{ where } C \text{ is a constant.}$$

   The resulting power series converges to $\int f(x)\,dx$ at all points in the interior of I.

Task 4 (Copy the book's solution for Sec 11.9, Example 5, pg 755):
Apply the first theorem and differentiate the geometric series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{ for } |x| < 1$$

to find a series representation for $\frac{1}{(1-x)^2}$ and give the interval of convergence of the new series.

<u>Task 4</u>. (Follow Sec 11.9, Example 8a, pg 756.)

Consider the geometric series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

i. Find a power series representation for $\frac{1}{1+x^3}$. (See similar Example 8a, top half).

Try to specify where the first Theorem (part 3) is used. What are f (t) and h (x) ?

ii. Evaluate $\int \frac{1}{1+x^3}\, dx$ as a power series and give the radius of convergence of the new series.

(See the similar solution of Example 8a, bottom half).

iii. Evaluate $\int_0^{0.1} \frac{1}{1+x^3}\,dx$ as a series. (Follow the top-half similar solution of Example 8b).

iv. Use the Alternating Series Remainder Theorem to find a bound on the error in approximating part (c) by adding up the first 4 terms of the series.
(Follow the bottom-half solution of Example 8b, pg 757. Use technology/ calculator).