Absolute Convergence

Recall

The geometric series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$ ________ to

The geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$ ________ to

The alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ __________ by

The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ __________ by

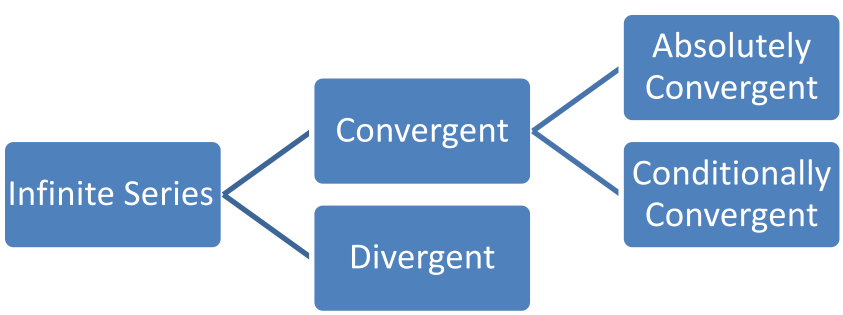
Above examples illustrate that removing the alternating signs in a convergent series may or may not result in a convergent series. Below terminology distinguishes these cases.

Absolute and Conditional Convergence

Definition Absolute and Conditional Convergence

Assume the infinite series $\sum a_n$ converges.

1. If ______________________, then we say that the series $\sum a_n$ **converges absolutely**.
2. If ______________________, then we say that the series $\sum a_n$ **converges conditionally**.



Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

Theorem 3 **Absolute Convergence Implies Convergence**

If ________________________________, then ________________________________.

Proof:

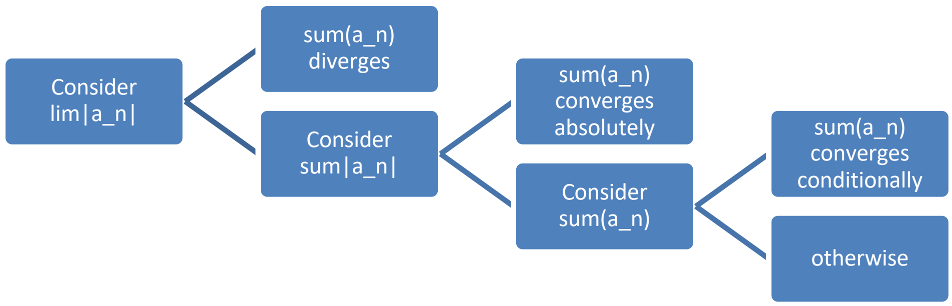
Procedure

Assume $\sum a_n$ is an alternating infinite series.

If $\lim_{n\to\infty}|a_n| \neq 0$, $\lim_{n\to\infty} a_n \neq 0$, $\sum a_n$ ______________________________________.

Otherwise, consider $\sum |a_n|$.

- If $\sum |a_n|$ converges, $\sum a_n$ ____.
- If $\sum |a_n|$ diverges, consider $\sum a_n$.
  - If $\{|a_n|\}$ is decreasing, $\sum a_n$ ______________________________________.



Example:

Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ diverges, converges absolutely, or converges conditionally.

Example:

Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ diverges, converges absolutely, or converges conditionally.

<u>Example</u>: (See also the solution of Example 3 in the textbook)

Determine whether $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ diverges, converges absolutely, or converges conditionally.