## Alternating Series

**Remainders in Alternating Series (pg 735)**

**Theorem** **Alternating Series Estimation Theorem**

Let $R_n = S - S_n$ be the **remainder** in approximating the value of a convergent alternating series $\sum_{n=1}^{\infty}(-1)^{n+1} b_n$ by the sum of its first $n$ terms. Then

$$|R_n| \le b_{n+1}.$$

In other words, the remainder is less than or equal to the magnitude of the first neglected term.

Example: Consider following convergent alternating series.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots \quad \ldots \quad = \ln 2$$ (See Exercise 36 pg 737)

a) Is the sum of sandwiched between any two consecutive partial sums?
b) Is it sandwiched between any two (non-consecutive) partial sums? Sketch the picture from Ex. 1 pg 734.

c) How many terms of $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ must be summed to be sure that the remainder is less than $10^{-4}$?

d) Find the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ correct to 2 decimal places.

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**<u>Def</u>:** A number r is called <u>rational</u> if

**<u>Application</u>:** Use the **Alternating Series Estimation Theorem** to prove that e is irrational.

<u>Fact we'll learn later in Sec 11.10</u>: 1/e =