
11.5 Alternating Series

Alternating Series Test (AST). If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots \quad b_n > 0$$

satisfies

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim_{n \rightarrow \infty} b_n = 0$

then the series is convergent.

Note: If either of the conditions of the AST are not met then you need to use a different test to determine whether the series is convergent or divergent.

1. **Example:** Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$$

Thinking about the problem:

Which test should I use to determine whether the series converges or diverges and why?

Have I seen a problem similar to this one before? If so, which test did I use?

To determine which test to use I write out a few terms of the series and notice that the sign of the successive terms of the series alternate between positive and negative. So I

will use the AST. I will let $b_n = \frac{1}{2n+1}$. At this point I need to check the conditions for the Alternating Series Test.

Doing the problem:

The problem asks whether the series converges or diverges. I note that the terms of the series alternate between positive and negative and I let $b_n = \frac{1}{2n+1}$ and apply

the Alternating Series Test (AST). Note that $b_n = \frac{1}{2n+1} > 0$ for all n . I can check

that $b_{n+1} \leq b_n$ for all n by either checking this explicitly, or letting $f(x) = \frac{1}{2x+1}$

and checking that $f'(x) \leq 0$ for all $x \geq 1$. I choose to check $f'(x)$. I compute that

$f'(x) = -\frac{2}{(2x+1)^2}$, which is negative for $x \geq 1$. Finally, $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{2n+1} = 0$.

So, by the AST, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$ converges.

Solutions should show all of your work, not just a single final answer.

2. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3 + 2}}$$

(a) Which test or tests could you use to determine whether the series converges or diverges?

(b) State the test you would use to decide whether the series converges or diverges.

(c) State b_n .

3. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$$