Names (please print):

Math 1152Q
Sec 11.4 Practice KEY

1. Determine whether each series converges or diverges.

$a.)\quad \sum_{n=1}^{\infty} \frac{\ln\ n}{n}$ $\qquad b.)\quad \sum_{n=4}^{\infty} \frac{1}{2^n - 9}$ (See also book answers: Sec 11.4 Examples 2 and 3).

Try Divergence Test: Both $\frac{\ln\ n}{n}$ and $\frac{1}{2^n-9}$ converge to 0. So the Divergence Test is inconclusive.

Try Limit Comparison Test (LCT):

For $a_n = \frac{\ln\ n}{n}$**:**

Try $b_n = \frac{1}{n}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\ln n}{n}\frac{n}{1}$$
$$= \lim_{n\to\infty} \ln n = \infty.$$

Since $\sum b_n$ diverges by the harmonic series test (p-series test), by the Limit Comparison Test, we conclude that $\sum a_n$ also diverges. (See also book's solution using Comparison Test. Sec 11.4 Example 2 pg 728-729.)

For $a_n = \frac{1}{2^n-9}$**:**

Try $b_n = \frac{1}{2^n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{2^n-9}\frac{2^n}{1}$$
$$= \lim_{n\to\infty} \frac{2^n}{2^n-9}$$
$$= \lim_{n\to\infty} \frac{1}{1-\frac{9}{2^n}}$$
$$= \frac{1}{1-(\lim_{n\to\infty}\frac{9}{2^n})} = 1, \quad \text{which is a positive number.}$$

We know that $\sum b_n$ converges by the geometric series test (or you can show this using the ratio test or root test). So, by the Limit Comparison test, we conclude that $\sum a_n$ also converges. (Warning: if $\lim_{n\to\infty} \frac{a_n}{b_n}$ had been ∞, we cannot conclude anything).

(For now, you can ignore this future topic) Try Ratio test:

For $a_n = \frac{\ln\ n}{n}$**:** you know that ratio tests will be *inconclusive* for terms involving only constants, log, and polynomials. See 1st page of "growth rates" notes: `https://egunawan.github.io/fall18/notes/notes11_6part3.pdf`

For $a_n = \frac{1}{2^n-9}$**:**

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^n-9}{2^{n+1}-9}$$
$$\stackrel{L}{=} \lim_{n\to\infty} \frac{\ln 2\ 2^n}{\ln 2\ 2^{n+1}}$$
$$= \frac{1}{2} < 1.$$

By the ratio test, $\sum_{n=4}^{\infty} \frac{1}{2^n-9}$ converges.

2. Determine whether each series is convergent or divergent.

$$i.)\quad \sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}} \qquad ii.)\quad \sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$$

(from Sec 11.4 Examples 1 and 4).

Try Divergence Test: Both $\dfrac{2n^2+3n}{\sqrt{5+n^5}}$ and $\dfrac{5}{2n^2+4n+3}$ converge to 0. The Divergence Test is inconclusive.

Try Limit Comparison Test (LCT):

For $a_n = \frac{2n^2+3n}{\sqrt{5+n^5}}$**:**

Try $b_n = \frac{1}{\sqrt{n}}$.

See textbook Sec 11.4 Example 4 page 730 for solution.

For $a_n = \frac{5}{2n^2+4n+3}$**:**

Try $b_n = \frac{1}{n^2}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{5}{2n^2+4n+3}\,\frac{n^2}{1} \\
&= \lim_{n\to\infty} \frac{5n^2}{2n^2+4n+3} \\
&= \lim_{n\to\infty} \frac{5}{2+\frac{4n}{n^2}+\frac{3}{n^2}} \\
&= \frac{5}{2+\lim_{n\to\infty}\frac{4}{n}+\lim_{n\to\infty}\frac{3}{n^2}} \\
&= \frac{5}{2+0+0} \\
&= \frac{5}{2}, \quad \text{which is a positive number.}
\end{aligned}$$

Since $\sum b_n$ converges by the p-series test, by the Limit Comparison Test, we conclude that $\sum a_n$ also converges.

(See also textbook Sec 11.4 Example 1 page 728 for solution using the comparison test.)

(For now, you can ignore this future topic) Try Ratio Test (for both series): You will find that the Ratio Test is inconclusive. Recall that ratio tests will be inconclusive for series that look like p-series, so you don't even need to try.