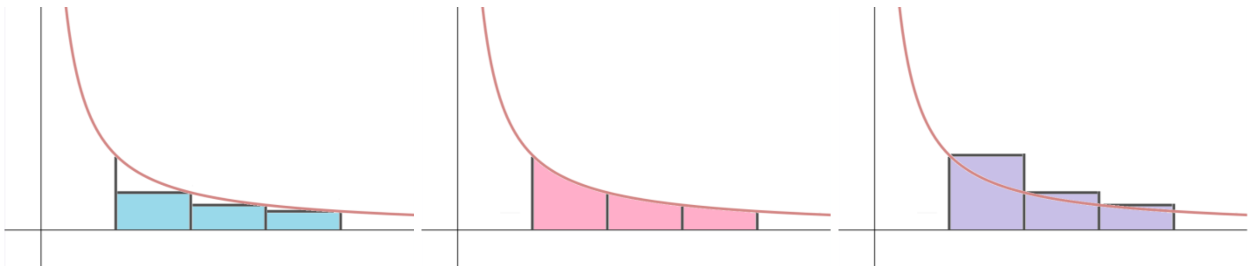
Estimating the Sum of a Series (pg 723)

We want to find an approximation to a convergent series $\sum a_n = S$. Any partial sum S_n is an approximation to S since $\lim_{n\to\infty} S_n = S$. But how good is such an approximation?

To find out, we need to estimate the size of the remainder

$$R_n = S - S_n = a_{n+1} + a_{n+2} + a_{n+3} + \quad \ldots$$

The remainder R_n is the error made when S_n is used as an approximation to S.



Remainder Estimate for the Integral Test (*) copy from pg 723

Suppose $f(k) = a_k$, where f is a continuous, positive, decreasing function for $x \geq n$ and $\sum a_n$ is convergent. If $R_n = S - S_n$, then

$$\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \leq R_n \leq \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}.$$

Carefully read and then copy Example 5 pg 723:

Consider the convergent infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Nobody has been able to find the exact sum.

- Example 5 part a: Approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by using the sum of the first 10 terms. Estimate the error involved in this approximation.

- Example 5 part b: How many terms are required to ensure that the sum is accurate to within 0.0005?

If we add S_n to each side of the inequalities in the previous result (*),

we get

Copy from Equation 3, page 724.

$$\underline{\qquad\qquad\qquad\qquad} \leq S \leq \underline{\qquad\qquad\qquad\qquad}. \quad (**)$$

Note: (**) gives a better estimate to the sum of the series than the partial sum S_n does.

<u>Read carefully then copy Example 6 (page 724)</u>:

Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^3}$. Given $S_{10} \approx 1.197532$, estimate the sum of the series.

Note (again): (**) gives a better estimate to the sum of the series than the partial sum S_n does.
Summarize the last paragraph on page 724.