Instruction: Fill in all blanks and examples.

Proof of Divergence Test

Divergence Test, Theorem 6, pg 713

Suppose the series $\sum_{k=1}^{\infty} a_k$ is convergent. Then $\lim_{k\to\infty} a_k = 0$.

Proof: Let S_n denote $S_n := a_1 + a_2 + \cdots + a_n$ for any $n = 1, 2, 3, \ldots$.

Then $S_{n-1} =$ ____________________ .

For example,

$$\begin{array}{rcccccccccc} S_5 & = & a_1 & + & a_2 & + & a_3 & + & a_4 & + & a_5 \\ S_4 & = & a_1 & + & a_2 & + & a_3 & + & a_4 & & \\ \hline S_5 - S_4 & = & & & & & & & & & \end{array} \quad -$$

In general,

$$\begin{array}{rcccccccccccc} S_n & = & a_1 & + & a_2 & + & a_3 & + & \ldots & + & a_{n-1} & + & a_n \\ S_{n-1} & = & a_1 & + & a_2 & + & a_3 & + & \ldots & + & a_{n-1} & & \\ \hline S_n - S_{n-1} & = & & & & & & & & & & & \end{array} \quad -$$

The above is an explanation that, in general,

$$a_n = \_\_\_\_\_\_\_\_\_\_\_\_ . \tag{1}$$

By assumption, $\sum_{n=1}^{\infty} a_n$ is convergent. By definition, __________ (see Def. 2, bottom of pg 708).

That is, for some real number S,

$$\_\_\_\_\_\_\_\_\_\_\_\_ = S \tag{2}$$

Since $n - 1 \to \infty$ as $n \to \infty$, we also have

$$\_\_\_\_\_\_\_\_\_\_\_\_ = S. \tag{3}$$

Therefore,

$$\begin{aligned} \lim_{n\to\infty} a_n &= \lim_{n\to\infty} (S_n - S_{n-1}) && \text{by (1)} \\ &= \lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} && \text{by Limit Laws for sequences (see pg 697)} \\ &= \_\_\_\_\_\_\_\_\_\_\_\_ && \text{by (2) and (3).} \end{aligned}$$

THE END OF PROOF

Harmonic Series Theorem, pg 713

The harmonic series $\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ is divergent/convergent (circle one).

Proof: