Sequences

Geometric Sequence

Visual

Geometric sequences have the property that each term is obtained by multiplying the previous term by a fixed constant, called the **ratio**.

$$\lim_{n\to\infty} r^n = \begin{cases} \_\_\_\_\_\_\_\_\_\_ & |r| < 1 \\ \_\_\_\_\_\_\_\_\_\_ & r = 1 \\ \_\_\_\_\_\_\_\_\_\_ & r = -1 \\ \_\_\_\_\_\_\_\_\_\_ & |r| > 1 \end{cases}$$

The sequence $\{r^n\}$ is **convergent** if $-1 < r \le 1$ and **divergent** for all other values of r.

Monotonic sequences

Definition

- A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geq 1$.
- A sequence $\{a_n\}$ is called **decreasing** if $a_n > a_{n+1}$ for all $n \geq 1$.
- A sequence $\{a_n\}$ is **monotonic** if it is either **increasing** or **decreasing**.

Example:

Show that the sequence $\left\{\frac{n}{n^2+1}\right\}$ is decreasing for $n > 1$.

Solution 1:

Solution 2:

Monotonic sequences continued

Definition

- A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geq 1$.
- A sequence $\{a_n\}$ is called **decreasing** if $a_n > a_{n+1}$ for all $n \geq 1$.
- A sequence $\{a_n\}$ is **monotonic** if it is either **increasing** or **decreasing**.

Another Example:

Make a prediction. Is the sequence { (n-1)/n } for n = 1, 2, 3, ... decreasing or increasing or neither?

Solution 1 (apply the definition directly):

Scratch work (for yourself): Polished solution:

Solution 2:

Consider the function f(x) = (x-1)/x.

Scratch work (for yourself): Polished solution:

Bounded sequences

Definition

- A sequence $\{a_n\}$ is said to be **bounded above** if

- A sequence $\{a_n\}$ is said to be **bounded below** if

- A sequence $\{a_n\}$ is said to be **bounded** if

Example:
What are some upper bounds and lower bounds of these sequences?

$\left\{\frac{n}{n^2+1}\right\}$ lower bounds: upper bounds:

{(n-1)/n} lower bounds: upper bounds:

Monotonic Sequence Theorem

If sequence $\{a_n\}$ is bounded and monotonic, then $\{a_n\}$ converges.

Example: The sequences $\left\{\frac{n}{n^2+1}\right\}$ and {(n-1)/n} are both monotonic and bounded, so by the monotonic sequence theorem, they are convergent.

Caution

Determine whether each statement is true/false. If false, give a counterexample. If true, explain.

1) If a sequence $\{a_n\}$ is bounded, then $\{a_n\}$ is convergent.

2) If a sequence $\{a_n\}$ is monotonic, then $\{a_n\}$ is convergent.

3) If a sequence $\{a_n\}$ is convergent, then $\{a_n\}$ is bounded.

4) If a sequence $\{a_n\}$ is convergent, then $\{a_n\}$ is monotonic.