Math 1152Q: Fall '18 **Week 2 In-class Practice, Friday, Sept 7**

Topics: Sec 11.1: Definition of convergence, ϵ, N proof, bounded sequences, decreasing/increasing sequences, Monotone Sequence Theorem.

> **Solution:** ANSWER KEY

1. (a) Let $\{b_n\}$ be a sequence. What does it mean to write $\lim_{n\to\infty} b_n = \infty$? Use Def 5, with M and N. (Answer: Copy from Sec 11.1 Def 5 page 697)

   (b) Given a sequence $\{a_n\}$, what does it mean to write $\lim_{n\to\infty} a_n = 4$ mean? Use the ϵ and N definition. (Answer: Copy from Sec 11.1 Def 2 pg 696)

   (c) What is a convergent sequence?

2. (a) What is a bounded sequence?

   (b) What is a monotonic sequence?

   (c) What can you say about a bounded monotonic sequence?

   (d) True or false? If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.

   > **Solution:** False. Counterexample: let $a_n = (-1)^n$ and $b_n = (-1)^{n+1}$ for $n = 1, 2, 3, \ldots$.

   (e) True or false? If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n\, b_n\}$ is divergent.

   > **Solution:** False. Counterexample: let $a_n = (-1)^n$ and $b_n = (-1)^{n+1}$ for $n = 1, 2, 3, \ldots$. or let $a_n = (-1)^n$ and $b_n = (-1)^n$ for $n = 1, 2, 3, \ldots$.

   (f) True or false? If $\{a_n\}$ and $\{b_n\}$ are convergent, then $\{a_n\, b_n\}$ is convergent.

   > **Solution:** True by Limit laws for sequences, see pg 697.

   (g) If $\{a_n\}$ is decreasing and $a_n > 0$ for all n, then $\{a_n\}$ is convergent.

   > **Solution:** True because the sequence is bounded above by the first term of the sequence and bounded below by 0. By monotonic sequence theorem, this sequence is convergent.

   (h) True or false (no justification needed for now)? $0.99999\cdots = 1$.

   > **Solution:** True. The two values are exactly equal. Will cover this in Sec 11.2.

   (i) True or false (no justification needed for now)? $0.333\ldots$ is close to $\frac{1}{3}$ but $0.333\cdots \neq \frac{1}{3}$.

   > **Solution:** False. The two values are exactly equal. Will cover this in Sec 11.2.

3. i.) (Graphing Review) Sketch each function. Label the asymptote/s and zero/s of the graph.

$g(x) = \dfrac{2x-4}{5x+8}$

Solution:

- $f(x) = \dfrac{2x-4}{5x+8}$ has a horizontal asymptote at $\frac{2}{5}$ because $\lim_{x\to\infty} f(x) = 2/5$.
- Note that $f(x) = \dfrac{2x-4}{5x+8}$ is not defined for $x = -8/5$ and that $-8/5$ is not a zero of the numerator $2x-4$. This tells us that $f(x)$ has a vertical asymptote at $x = -\dfrac{8}{5}$.
- Definition: The graph of $y = f(x)$ is said to have a vertical asymptote $x = a$ if $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$.
- To figure out whether your graph approaches $+\infty$ or $-\infty$ to the right, plug in a number bigger than a and estimate whether it looks very large (positive) or very large (negative).

ii.) Fill in the blanks with either the sign $\leq$ or $\geq$.

$$\frac{5n!}{2^n} \quad \_\_\_\_\_ \quad \left(\frac{1}{2}\right)^n \quad \text{for all } n \geq 1$$
$$\frac{n-1}{7n+4} \quad \_\_\_\_\_ \quad \frac{1}{7} \quad \text{for all } n \geq 1$$
$$\frac{n+1}{7n-4} \quad \_\_\_\_\_ \quad \frac{1}{7} \quad \text{for all } n \geq 1$$

Solution:

$$\frac{5n!}{2^n} \geq \left(\frac{1}{2}\right)^n \quad \text{for all } n \geq 1$$
$$\frac{n-1}{7n+4} \leq \frac{1}{7} \quad \text{for all } n \geq 1$$
$$\frac{n+1}{7n-4} \geq \frac{1}{7} \quad \text{for all } n \geq 1$$

iii.) The sequence $a_k = \dfrac{2k+4}{5k-8}$ converges to 2/5. For any number ϵ where $0 < \epsilon < 1$, choose N so that if $k > N$, then $\left|\dfrac{2}{5} - a_k\right| < \epsilon$.